Chapter 5 Quadratic Equations in One Variable Ex 5.2

Solve the following equations (1 to 24) by factorization:

Question 1.
(i) $4x^2 = 3x$
(ii) $x^2 - 5x^2 = 0$
Solution:

(i) $4x^2 = 3x$

$x(4x - 3) = 0$

Either $x = 0$,

or $4x - 3 = 0$, then $4x = 3 \Rightarrow x = \frac{3}{4}$

$\therefore x = 0, \frac{3}{4}$

(ii) $\frac{x^2 - 5x}{2} = 0$

$x^2 - 5x = 0$

$\Rightarrow \quad x(x - 5) = 0$

Either $x = 0$ or $x - 5 = 0$, then $x = 5$

Hence $x = 0, 5$

Question 2.
(i) $(x - 3)(2x + 5) = 0$
(ii) $x(2x + 1) = 6$

Solution:

(i) $(x - 3)(2x + 5) = 0$

Either $x - 3 = 0$,

Then $x = 3$

or $2x + 5 = 0$ then $2x = -5 \Rightarrow x = \frac{-5}{2}$

Hence $x = 3, = \frac{-5}{2}$

**(ii)** $x(2x + 1) = 6$ **(1993)**

$x(2x + 1) = 6$

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0 \Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

Either $x + 2 = 0$, then $x = -2$

or $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \frac{3}{2}$

Hence $x = -2, \frac{3}{2}$

Question 3.

(i) $x^2 - 3x - 10 = 0$

(ii) $x(2x + 5) = 3$

Solution:

(i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

sEither $x - 5 = 0$, then $x = 5$

or $x + 2 = 0$, then $x = -2$

Hence $x = 5, -2$

**(ii)** $x(2x + 5) = 3$ **(1994)**

$x(2x + 5) = 3$

$\Rightarrow 2x^2 + 5x - 3 = 0 \Rightarrow 2x^2 + 6x - x - 3 = 0$

$\Rightarrow 2x(x + 3) - 1(x + 3) = 0$

$\Rightarrow (x + 3)(2x - 1) = 0$

Either $x + 3 = 0$, then $x = -3$

or $2x - 1 = 0$, then $2x = 1 \Rightarrow x = \frac{1}{2}$

$\therefore x = -3, \frac{1}{2}$

Question 4.

(i) $3x^2 - 5x - 12 = 0$

(ii) $21x^2 - 8x - 4 = 0$

Solution:

(i) $3x^2 - 5x - 12 = 0$

$\Rightarrow 3x^2 - 9x + 4x - 12 = 0$

$\Rightarrow 3x(x - 3) + 4(x - 3) = 0$

$\Rightarrow (x - 3)(3x + 4) = 0$

Either $x - 3 = 0$, then $x = 3$

or $3x + 4 = 0$, then $3x = -4 \Rightarrow x = \frac{-4}{3}$

Hence $x = 3, \frac{-4}{3}$

**(ii)** $21x^2 - 8x - 4 = 0$ **(1990)**

$21x^2 - 8x - 4 = 0$

$\Rightarrow 21x^2 - 14x + 6x - 4 = 0$

$\Rightarrow 7x(3x - 2) + 2(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 2) = 0$

Either $3x - 2 = 0$, then $3x = 2 \Rightarrow x = \frac{2}{3}$

or $7x + 2 = 0$, then $7x = -2 \Rightarrow x = \frac{-2}{7}$

Hence $x = \frac{2}{3}, \frac{-2}{7}$

Question 5.

(i) $3x^2 = x + 4$

(ii) $x(6x - 1) = 35$

Solution:

(i) $3x^2 = x + 4$

$\Rightarrow 3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow (3x - 4)(x + 1) = 0$

Either $3x - 4 = 0$, then $3x = 4 \Rightarrow x = \frac{4}{3}$

or $x + 1 = 0$, then $x = -1$

Hence $x = \frac{4}{3}, -1$

**(ii)** $x(6x - 1) = 35$ **(1991)**

$x(6x - 1) = 35$

$\Rightarrow 6x^2 - x - 35 = 0$

$\Rightarrow 6x^2 - 15x + 14x - 35 = 0$

$\Rightarrow 3x(2x - 5) + 7(2x - 5) = 0$

$\Rightarrow (2x - 5)(3x + 7) = 0$

Either $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \frac{5}{2}$

or $3x + 7 = 0$, then $3x = -7 \Rightarrow x = \frac{-7}{3}$

Hence $x = \frac{5}{2}, \frac{-7}{3}$

Question 6.

(i) $6p^2 + 11p - 10 = 0$

(ii) $23x2-13x=1$

Solution:

(i) $6p^2 + 11p - 10 = 0$

$\Rightarrow 6p^2 + 15p - 4p - 10 = 0$

$\Rightarrow 3p(2p + 5) - 2(2p + 5) = 0$

$(2p + 5)(3p - 2) = 0$

Either $2p + 5 = 0$, then $2p = -5 \Rightarrow p = \frac{-5}{2}$

or $3p - 2 = 0$, then $3p = 2 \Rightarrow p = \frac{2}{3}$

Hence $p = \frac{-5}{2}, \frac{2}{3}$

**(ii)** $\frac{2}{3}x^2 - \frac{1}{3}x = 1$

$\frac{2}{3}x^2 - \frac{1}{3}x = 1$

$\Rightarrow 2x^2 - x = 3 \Rightarrow 2x^2 - x - 3 = 0$

$\Rightarrow 2x^2 - 3x + 2x - 3 = 0$

$\Rightarrow x(2x - 3) + 1(2x - 3) = 0$

$\Rightarrow (2x - 3)(x + 1) = 0$

Either $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \frac{3}{2}$

or $x + 1 = 0$, then $x = -1$

Hence $x = 3/2, -1$

Question 7.

(i) $(x - 4)^2 + 5^2 = 13^2$

(ii) $3(x - 2)^2 = 147$

Solution:

(i) $(x - 4)^2 + 5^2 = 13^2$

$x^2 - 8x + 16 + 25 = 169$

$x^2 - 8x + 41 - 169 = 0 \Rightarrow x^2 - 8x - 128 = 0$

$\Rightarrow x^2 - 16x + 8x - 128 = 0$

$\Rightarrow x(x - 16) + 8(x - 16) = 0$

$\Rightarrow (x - 16)(x + 8) = 0$

Either $x - 16 = 0$, then $x = 16$

or $x + 8 = 0$, then $x = -8$

Hence $x = 16, -8$

**(ii)** $3(x - 2)^2 = 147$

$3(x - 2)^2 = 147$

$3(x^2 - 4x + 4) = 147$

$\Rightarrow 3x^2 - 12x + 12 - 147 = 0$

$\Rightarrow 3x^2 - 12x - 135 = 0$

$\Rightarrow x^2 - 4x - 45 = 0$ (dividing by 3)

$\Rightarrow x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x - 9) + 5(x - 9) = 0$

$\Rightarrow (x - 9)(x + 5) = 0$

Either $x - 9 = 0$, then $x = 9$

or $x + 5 = 0$, then $x = -5$

Hence $x = 9, -5$

Question 8.

(i) $17(3x - 5)^2 = 28$

(ii) $3(y^2 - 6) = y(y + 7) - 3$

Solution:

(i) $\frac{1}{7}(3x - 5)^2 = 28$

$(3x - 5)^2 = 28 \times 7$

$\Rightarrow 9x^2 - 30x + 25 = 196$

$\Rightarrow 9x^2 - 30x + 25 - 196 = 0$

$\Rightarrow 9x^2 - 30x - 171 = 0$

$\Rightarrow 3x^2 - 10x - 57 = 0$ (Dividing by 3)

$\Rightarrow 3x^2 - 19x + 9x - 57 = 0$

$\Rightarrow x(3x - 19) + 3(3x - 19) = 0$

$\Rightarrow (3x - 19)(x + 3) = 0$

Either $3x - 19 = 0$, then $3x = 19$

$\Rightarrow x = \frac{19}{3}$ or $x + 3 = 0$, then $x = -3$

Hence $x = \frac{19}{3}, -3$

**(ii)** $3(y^2 - 6) = y(y + 7) - 3$

$3(y^2 - 6) = y(y + 7) - 3$

$\Rightarrow 3(y^2 - 6) = y^2 + 7y - 3$

$\Rightarrow 3y^2 - 18 = y^2 + 7y - 3$

$\Rightarrow 3y^2 - y^2 - 7y - 18 + 3 = 0$

$\Rightarrow 2y^2 - 7y - 15 = 0$

$\Rightarrow 2y^2 - 10y + 3y - 15 = 0$

$2y(y - 5) + 3(y - 5) = 0$

$\Rightarrow (y - 5)(2y + 3) = 0$

Either $y - 5 = 0$, then $y = 5$

or $2y + 3 = 0$, then $2y = -3 \Rightarrow y = -3/2$

Hence $y = -3/2, 5$

Question 9.

$x^2 - 4x - 12 = 0$, when $x \in N$

Solution:

$x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x - 6) + 2(x - 6) = 0$

$\Rightarrow (x - 6)(x + 2) = 0$

Either $x - 6 = 0$, then $x = 6$

or $x + 2 = 0$, then $x = -2$

But -2 is not a natural number

$\therefore x = 6$

Question 10.

$2x^2 - 8x - 24 = 0$ when $x \in I$

Solution:

$2x^2 - 8x - 24 = 0$

$\Rightarrow x^2 - 4x - 12 = 0$ (Dividing by 2)

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x - 6) + 2(x - 6) = 0$

$\Rightarrow (x - 6)(x + 2) = 0$

Either $x - 6 = 0$, then, $x = 6$

or $x + 2 = 0$, then $x = -2$

Hence $x = 6, -2$

Question 11.

$5x^2 - 8x - 4 = 0$ when $x \in Q$

Solution:

$5x^2 - 8x - 4 = 0$

$\because 5 \times (-4) = -20$

$-20 = -10 + 2$

$-8 = -10 + 2$

$\Rightarrow 5x^2 - 10x + 2x - 4 = 0$

$\Rightarrow 5x(x-2) + 2(x-2) = 0$

$\Rightarrow (x-2)(5x+2) = 0$

(Zero Product Rule)

Either $x - 2 = 0$, then $x = 2$

or $5x + 2 = 0$, then $5x = -2 \Rightarrow x = -\frac{2}{5}$

$\therefore x = 2, -\frac{2}{5}$

Question 12.

$2x^2 - 9x + 10 = 0$,when

(i) $x \in N$

(ii) $x \in Q$

Solution:

$2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 4x - 5x + 10 = 0$

$\Rightarrow 2x(x-2) - 5(x-2) = 0$

$\Rightarrow (x-2)(2x-5) = 0$

Either $x - 2 = 0$, then $x = 2$,

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \frac{5}{2}$

(*i*) when $x \in \mathbf{N}$, then $x = 2$

(*ii*) when $x \in \mathbf{Q}$, then $x = 2, \frac{5}{2}$

Question 13.

(i) $a^2x^2 + 2ax + 1 = 0$, $a \neq 0$

(ii) $x^2 - (p+q)x + pq = 0$

Solution:

(i) $a^2x^2 + 2ax + 1 = 0$

$\Rightarrow a^2x^2 + ax + ax + 1 = 0$

$\Rightarrow ax(ax + 1) + 1(ax + 1) = 0$

$\Rightarrow (ax + 1)(ax + 1) = 0 \Rightarrow (ax + 1)^2 = 0$

$\therefore ax + 1 = 0 \Rightarrow ax = -1 \quad x = -\frac{1}{a}$

Hence $x = -\frac{1}{a}, -\frac{1}{a}$

**(ii)** $x^2 - (p + q)x + pq = 0$

$x^2 - (p + q)x + pq = 0$

$x^2 - px - qx + pq = 0$

$x(x - p) - q(x - p) = 0$

$\Rightarrow (x - p)(x - q) = 0$

Either $x - p = 0$, then $x = p$,

or $x - q = 0$, then $x = q$

Hence $x = p, q$

Question 14.

$a^2x^2 + (a^2 + b^2)x + b^2 = 0, a \neq 0$

Solution:

$a^2x^2 + (a^2 + b^2)x + b^2 = 0$

$\Rightarrow a^2x(x + 1) + b^2(x + 1) = 0$

$\Rightarrow (x + 1)(a^2x + b^2) = 0.$

$\Rightarrow (x + 1) = 0$, then $x = -1$

or $a^2x + b^2 = 0$, then $a^2x = -b^2 \Rightarrow x = \frac{-b^2}{a^2}$

Hence $x = -1, \frac{-b^2}{a^2}$

Question 15.

(i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Solution:

(i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$[\because \sqrt{3} \times 7\sqrt{3} = 7 \times 3 = 21]$

$\Rightarrow \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$\Rightarrow (x + \sqrt{3})(\sqrt{3}x + 7) = 0$

Either $x + \sqrt{3} = 0$, then $x = -\sqrt{3}$

or $\sqrt{3}x + 7 = 0$, then $\sqrt{3}x = -7$

$\Rightarrow x = \frac{-7}{\sqrt{3}} \quad \Rightarrow x = \frac{-7 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{-7\sqrt{3}}{3}$

Hence $x = -\sqrt{3}, -\frac{7\sqrt{3}}{3}$

**(ii)** $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\left\{4\sqrt{3} \times (-2\sqrt{3}) = 8 \times (-3) = -24\right\}$

$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$

Either $\sqrt{3}x + 2 = 0$, then $\sqrt{3}x = -2$

$\Rightarrow x = -\frac{2}{\sqrt{3}}$

$\Rightarrow x = -\frac{-2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = -\frac{-2\sqrt{3}}{3}$

or $4x - \sqrt{3} = 0$, then $4x = \sqrt{3}$

$\Rightarrow x = \frac{\sqrt{3}}{4}$

Hence $x = \frac{-2\sqrt{3}}{3}, \frac{\sqrt{3}}{4}$

Question 16.

(i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

(ii) $x + \frac{1}{x} = 2\frac{1}{20}$

Solution:

(i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x(x - 1) - \sqrt{2}(x - 1) = 0$

$\Rightarrow (x - 1)(x - \sqrt{2}) = 0$

Either $x - 1 = 0$, then $x = 1$

or $x - \sqrt{2} = 0$, then $x = \sqrt{2}$

Hence $x = 1, \sqrt{2}$

**(ii)** $x + \frac{1}{x} = 2\frac{1}{20}$

$\frac{x^2 + 1}{x} = \frac{41}{20} \Rightarrow 20x^2 + 20 = 41x$

$\Rightarrow 20x^2 - 41x + 20 = 0$

$\Rightarrow 20x^2 - 16x - 25x + 20 = 0$

$\Rightarrow 4x(5x - 4) - 5(5x - 4) = 0$

$\Rightarrow (5x - 4)(4x - 5) = 0$

Either $5x - 4 = 0$, then $5x = 4 \Rightarrow x = \frac{4}{5}$

or $4x - 5 = 0$, then $4x = 5 \Rightarrow x = \frac{5}{4}$

Hence $x = \frac{4}{5}, \frac{5}{4}$

Question 17.

(i) $2x^2 - 5x + 2 = 0, x \neq 0$

(ii) $\frac{x}{2} - \frac{15}{x} - 3 = 10$

Solution:

(i) $\frac{2}{x^2} - \frac{5}{x} + 2 = 0, x \neq 0$

$\Rightarrow 2 - 5x + 2x^2 = 0$

$\Rightarrow 2x^2 - 5x + 2 = 0$ $\left\{\begin{array}{l} \because 2 \times 2 = 4 \\ 4 = -4 \times (-1) \\ -5 = -4 - 1 \end{array}\right\}$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 2)(2x - 1) = 0$

Either $x - 2 = 0$, then $x = 2$

or $2x - 1 = 0$, then $2x = 1 \Rightarrow x = \frac{1}{2}$

$\therefore x = 2, \frac{1}{2}$

(ii) $\frac{x^2}{15} - \frac{x}{3} - 10 = 0$

$\Rightarrow x^2 - 5x - 150 = 0$

$\left\{\begin{array}{l} \because -150 = -15 \times 10 \\ -5 = -15 + 10 \end{array}\right\}$

$\Rightarrow x^2 - 15x + 10x - 150 = 0$

$\Rightarrow x(x - 15) + 10(x - 15) = 0$

$\Rightarrow (x - 15)(x + 10) = 0$

Either $x - 15 = 0$, then $x = 15$

or $x + 10 = 0$, then $x = -10$

$\therefore x = 15, -10$

Question 18.

(i) $3x-8x=2$

(ii) $x+2x+3=2x-33x-7$

Solution:

(i) $3x - \frac{8}{x} = 2$

$\frac{3x^2-8}{x} = 2$

$\Rightarrow 3x^2 - 8 = 2x$

$\Rightarrow 3x^2 - 2x - 8 = 0$

$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$

$\Rightarrow 3x(x-2) + 4(x-2) = 0$

$\Rightarrow (x-2)(3x+4) = 0$

Either $x - 2 = 0$, then $x = 2$

or $3x + 4 = 0$, then $3x = -4 \Rightarrow x = \frac{-4}{3}$

Hence $x = 2, -4/3$

(ii) $\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$

$(x + 2)(3x - 7) = (2x - 3)(x + 3)$

$\Rightarrow 3x^2 - 7x + 6x - 14 = 2x^2 + 6x - 3x - 9$

$\Rightarrow 3x^2 - x - 14 = 2x^2 + 3x - 9$

$\Rightarrow 3x^2 - x - 14 - 2x^2 - 3x + 9 = 0$

$\Rightarrow x^2 - 4x - 5 = 0 \Rightarrow x^2 - 5x + x - 5 = 0$

$x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (x - 5)(x + 1) = 0$

Either $x - 5 = 0$, then $x = 5$

or $x + 1 = 0$, then $x = -1$

Hence $x = 5, -1$

Question 19.

(i) $8x+3-32-x=2$

(ii) $xx-1+x-1x=2212$

Solution:

(i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

$\frac{16-8x-3x-9}{(x+3)(2-x)} = 2$

$$\Rightarrow \frac{-11x + 7}{2x - x^2 + 6 - 3x} = 2$$

$\Rightarrow -11x + 7 = 4x - 2x^2 + 12 - 6x$

$\Rightarrow -11x + 7 - 4x + 2x^2 - 12 + 6x = 0$

$\Rightarrow 2x^2 - 9x - 5 = 0$

$\Rightarrow 2x^2 - 10x + x - 5 = 0$

$\Rightarrow 2x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (x - 5)(2x + 1) = 0$

Either $x - 5 = 0$, then $x = 5$

or $2x + 1 = 0$, then $2x = -1 \Rightarrow x = -\frac{1}{2}$

Hence $x = 5, -\frac{1}{2}$

**(ii)** $\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$

$\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$

$\frac{x}{x-1}+\frac{x-1}{x}=\frac{5}{2}$

$\Rightarrow \frac{x^2+x^2-2x+1}{x(x-1)}=\frac{5}{2}$

$\Rightarrow \frac{2x^2-2x+1}{x^2-x}=\frac{5}{2}$

$\Rightarrow 4x^2-4x+2=5x^2-5x$

$\Rightarrow 4x^2-4x+2-5x^2+5x=0$

$\Rightarrow -x^2+x+2=0 \Rightarrow x^2-x-2=0$

$\Rightarrow x^2-2x+x-2=0$

$\Rightarrow x(x-2)+1(x-2)=0$

$\Rightarrow (x-2)(x+1)=0$

Either $x-2=0$, then $x=2$

or $x+1=0$, then $x=-1$

Hence $x=2,-1$

Question 20.

(i) $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$

(ii) $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$

Solution:

(i) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

$\frac{x^2+x^2+2x+1}{x(x+1)} = \frac{34}{15}$

$$\Rightarrow \quad \frac{2x^2+2x+1}{x^2+x} = \frac{34}{15}$$

$$\Rightarrow \quad 30x^2 + 30x + 15 = 34x^2 + 34x$$

$$\Rightarrow \quad 30x^2 + 30x + 15 - 34x^2 - 34x = 0$$

$$\Rightarrow \quad -4x^2 - 4x + 15 = 0$$

$$\Rightarrow \quad 4x^2 + 4x - 15 = 0$$

$$\Rightarrow \quad 4x^2 + 10x - 6x - 15 = 0$$

$$\Rightarrow \quad 2x(2x + 5) - 3(2x + 5) = 0$$

$$\Rightarrow \quad (2x + 5)(2x - 3) = 0$$

Either $2x + 5 = 0$, then $2x = -5 \Rightarrow x = \frac{-5}{2}$

or $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \frac{3}{2}$

Hence $x = \frac{-5}{2}, \frac{3}{2}$

**(ii)** $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$$

$$\Rightarrow \frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)}=3$$

$$\Rightarrow \frac{x^2+2x+x+2+x^2-x-2x+2}{x^2+2x-x-2}$$

$$\Rightarrow \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2}=\frac{3}{1}$$

$\Rightarrow 2x^2 + 4 = 3x^2 + 3x - 6$

$\Rightarrow 2x^2 + 4 - 3x^2 - 3x + 6 = 0$

$\Rightarrow -x^2 - 3x + 10 = 0$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x + 5) - 2(x + 5) = 0$

$\Rightarrow (x + 5)(x - 2) = 0$

Either $x + 5 = 0$, then $x = -5$

or $x - 2 = 0$, then $x = 2$

Hence $x = -5, 2$

Question 21.

(i) $1x-3-1x+5=16$

(ii) $x-3x+3+x+3x-3=212$

Solution:

(i) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

$$\frac{x+5-x+3}{(x-3)(x+5)} = \frac{1}{6} \Rightarrow \frac{8}{x^2+2x-15} = \frac{1}{6}$$

$\Rightarrow x^2 + 2x - 15 = 48$

$\Rightarrow x^2 + 2x - 15 - 48 = 0$

$\Rightarrow x^2 + 2x - 63 = 0 \Rightarrow x^2 + 9x - 7x - 63 = 0$

$\Rightarrow x(x + 9) - 7(x + 9) = 0$

$\Rightarrow (x + 9)(x - 7) = 0$

Either $x + 9 = 0$, then $x = -9$

or $x - 7 = 0$, then $x = 7$

Hence $x = -9, 7$

**(ii)** $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$

$$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$$

Put $\frac{x-3}{x+3} = a$, then $\frac{x+3}{x-3} = \frac{1}{a}$

$\therefore a + \frac{1}{a} = \frac{5}{2}$

$2a^2 + 2 = 5a$

$\Rightarrow 2a^2 - 5a + 2 = 0 \Rightarrow 2a^2 - a - 4a + 2 = 0$

$\Rightarrow a(2a - 1) - 2(2a - 1) = 0$

$\Rightarrow (2a - 1)(a - 2) = 0$

Either $2a - 1 = 0$, then $a = \frac{1}{2}$

or $a - 2 = 0$, then $a = 2$

(a) When $a = \frac{1}{2}$, then

$$\frac{x-3}{x+3} = \frac{1}{2} \Rightarrow 2x - 6 = x + 3$$

$\Rightarrow 2x - x = 3 + 6 \Rightarrow x = 9$

(b) When $a = 2$, then

$$\frac{x-3}{x+3} = \frac{2}{1}$$

$2x + 6 = x - 3 \Rightarrow 2x - x = -3 - 6$

$\Rightarrow x = -9$

$\therefore x = 9, -9$

Question 22.

(i) $aax-1+bbx-1=a+b, a+b\neq0, ab\neq0$

(ii) $12a+b+2x=12a+1b+12x$

Solution:

(i) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$

$\Rightarrow \left(\frac{a}{ax-1} - b\right) + \left(\frac{b}{bx-1} - a\right) = 0$

$$\Rightarrow \frac{a - abx + b}{(ax-1)} + \frac{b - abx + a}{(bx-1)} = 0$$

$$\Rightarrow (a + b - abx)\left[\frac{1}{ax-1} + \frac{1}{bx-1}\right] = 0$$

$$\Rightarrow (a + b - abx)\left[\frac{bx - 1 + ax - 1}{(ax-1)(bx-1)}\right] = 0.$$

$$\Rightarrow \frac{(a + b - abx)(ax + bx - 2)}{(ax-1)(bx-1)} = 0$$

$\Rightarrow (a + b - abx)(ax + bx - 2) = 0$

$\Rightarrow$ Either $a + b - abx = 0$, then $a + b = abx$

$$x = \frac{a+b}{ab}$$

or $ax + bx - 2 = 0$, then $x(a + b) = 2$

$$x = \frac{2}{a+b}$$

Hence $x = \frac{a+b}{ab}, \frac{2}{a+b}$

(ii) $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow \frac{2x-(2a+b+2x)}{(2a+b+2x)2x} = \frac{b+2a}{2ab}$

$\Rightarrow \frac{-(2a+b)}{(2a+b+2x)2x} = \frac{(2a+b)}{2ab}$

$\Rightarrow \frac{-1}{(2a+b+2x)2x} = \frac{1}{2ab}$

$\Rightarrow -2ab = (2a + b + 2x)2x$

$\Rightarrow 4ax + 2xb + 4x^2 = -2ab$

$\Rightarrow 4x^2 + 2bx + 4ax + 2ab = 0$

$\Rightarrow 2x(2x + b) + 2a\,(2x + b) = 0$

$\Rightarrow (2x + 2a)(2x + b) = 0$

$\Rightarrow 2x + 2a = 0$ or $2x + b = 0$

$x = -a$ or $x = \frac{-b}{2}$

Hence, the roots of the given equation are

$-a$ and $\frac{-b}{2}$.

Question 23.

$1x+6+1x-10=3x-4$

Solution:

$\frac{1}{x+6} + \frac{1}{x-10} = \frac{3}{x-4}$

$\Rightarrow \frac{x-10+x+6}{(x+6)(x-10)} = \frac{3}{x-4}$

$$\Rightarrow \frac{2x-4}{(x+6)(x-10)} = \frac{3}{x-4}$$

$$\Rightarrow (2x-4)(x-4) = 3(x+6)(x-10)$$

$$\Rightarrow 2x^2 - 8x - 4x + 16 = 3(x^2 - 4x - 60)$$
$$\Rightarrow 2x^2 - 8x - 4x + 16 = 3x^2 - 12x - 180$$
$$\Rightarrow 2x^2 - 12x + 16 - 3x^2 + 12x + 180 = 0$$
$$\Rightarrow -x^2 + 196 = 0 \quad \Rightarrow x^2 - 196 = 0$$
$$\Rightarrow (x)^2 - (14)^2 = 0 \quad \Rightarrow (x+14)(x-14) = 0$$

Either $x + 14 = 0$, then $x = -14$

or $x - 14 = 0$, then $x = 14$

$\therefore$ $x = \mathbf{14, -14}$

Question 24.

(i) $3x+4-----\sqrt{}=x$

(ii) $x(x-7)--------\sqrt{}=32-\sqrt{}$

Solution:

(i) $\sqrt{3x+4} = x$

Squaring on both sides

$3x + 4 = x^2 \Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow x^2 - 4x + x - 4 = 0, \Rightarrow x(x-4) + 1(x-4) = 0$

$\Rightarrow (x-4)(x+1) = 0$

Either $x - 4 = 0$, then $x = 4$

or $x + 1 = 0$, then $x = -1$

$\therefore x = 4, -1$

Check (*i*) If $x = 4$, then

L.H.S. $= \sqrt{3x+4} = \sqrt{3\times4+4}$

$= \sqrt{12+4} = \sqrt{16} = 4$

R.H.S. $= x = 4$ $\quad \therefore$ L.H.S. = R.H.S.

Hence $x = 4$ is its root

(*ii*) If $x = -1$, then

L.H.S. $= \sqrt{3\times(-1)+4} = \sqrt{-3+4} = \sqrt{1} = 1$

R.H.S. $= x = -1$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore x = -1$ is not its root, Hence $x = 4$

**(ii)** $\sqrt{x(x-7)} = 3\sqrt{2}$

$\sqrt{x(x-7)} = 3\sqrt{2}$ , Squaring both sides,

$x(x-7) = 9 \times 2 \Rightarrow x^2 - 7x = 18$

$\Rightarrow \quad x^2 - 7x - 18 = 0,$

$\Rightarrow \quad x^2 - 9x + 2x - 18 = 0$

$\Rightarrow \quad x(x-9) + 2(x-9) = 0$

$\Rightarrow \quad (x-9)(x+2) = 0$

Either $\quad x - 9 = 0$, then $x = 9$

or $\quad x + 2 = 0$, then $x = -2$

$\therefore x = 9, -2$

Check : (*i*) If $x = 9$, then

L.H.S. $= \sqrt{x(x-7)} = \sqrt{9(9-7)}$

$= \sqrt{9\times2} = \sqrt{18} = \sqrt{9\times2} = 3\sqrt{2}$ = R.H.S.

$x = 9$ is a root

(*ii*) If $x = -2$. then

L.H.S. $= \sqrt{x(x-7)} = \sqrt{-2(-2-7)}$

$= \sqrt{-2\times-9} = \sqrt{18} = \sqrt{9\times2} = 3\sqrt{2}$ = R.H.S

$\therefore x = -2$ is also its root

Hence $x = 9, -2$

Question 25.

Use the substitution y = 3x + 1 to solve for x : $5(3x + 1)^2 + 6(3x + 1) - 8 = 0$

Solution:

$y = 3x + 1$

Now, $5(3x + 1)^2 + 6(3x + 1) - 8 = 0$

Substituting the value of $3x + 1$, we get

$5y^2 + 6y - 8 = 0 \Rightarrow 5y^2 + 10y - 4y - 8 = 0$

$$\left\{\begin{array}{l}\because 5\times(-8)=-40\\ \therefore -40=10\times(-4)\\ \quad 6=10-4\end{array}\right\}$$

$\Rightarrow 5y(y + 2) - 4(y + 2) = 0 \Rightarrow (y + 2)(5y - 4) = 0$

Either $y + 2 = 0$, then $y = -2$

or $5y - 4 = 0$ then $5y = 4 \Rightarrow y = \frac{4}{5}$

(*i*) If $y = -2$, then

$3x + 1 = -2 \Rightarrow 3x = -2 - 1$

$\Rightarrow 3x = -3 \Rightarrow x = \frac{-3}{3} = -1$

(*ii*) If $y = \frac{4}{5}$, then

$3x + 1 = \frac{4}{5} \Rightarrow 3x = \frac{4}{5} - 1 = \frac{-1}{5}$

$\Rightarrow x = \frac{-1}{5} \times \frac{1}{3} = \frac{-1}{15}$

Hence $x = -1, \frac{-1}{15}$

Question 26.

Find the values of x if p + 1 =0 and $x^2 + px - 6 = 0$

Solution:

$p + 1 = 0$, then $p = -1$

Substituting the value of p in the given quadratic equation

$x^2 + (-1)x - 6 = 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

Either $x - 3 = 0$, then $x = 3$

or $x + 2 = 0$, then $x = -2$

Hence $x = 3, -2$

Question 27.

Find the values of x if $p + 7 = 0$, $q - 12 = 0$ and $x^2 + px + q = 0$,

Solution:

$p + 7 = 0$, then $p = -7$

and $q - 12 = 0$, then $q = 12$

Substituting the values of p and q in the given quadratic equation,

$x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 3x - 4x + 12 = 0$

$\Rightarrow x(x - 3) - 4(x - 3) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

Either $x - 3 = 0$, then $x = 3$

or $x - 4 = 0$, then $x = 4$

Hence $x = 3, 4$

Question 28.

If $x = p$ is a solution of the equation $x(2x + 5) = 3$, then find the value of p.

Solution:

Given, x = p and x(2x + 5) = 3

Substituting the value of p, we get

p(2p + 5) = 3

$\Rightarrow 2p^2 + 5p - 3 = 0$

$\Rightarrow 2p^2 + 6p - p - 3 = 0$

$$\left\{\begin{array}{l}\because 2\times(-3)=-6\\ \therefore -6=6\times(-1)\\ 5=6-1\end{array}\right\}$$

$\Rightarrow 2p(p + 3) - 1(p + 3) = 9 \Rightarrow (p + 3)(2p - 1) = 0$

Either $p + 3 = 0$, then $p = -3$

or $2p - 1 = 0$, then $2p = 1 \Rightarrow p = \frac{1}{2}$

$\therefore\ p = \frac{1}{2}, -3$