Chapter 7 Ratio and Proportion Ex 7.1

Question 1.
An alloy consists of $27\frac{1}{2}$ kg of copper and $2\frac{3}{4}$ kg of tin. Find the ratio by weight of tin to the alloy
Solution:

Copper = $27\frac{1}{2}$ kg = $\frac{55}{2}$ kg,

Tin = $2\frac{3}{4}$ kg = $\frac{11}{4}$ kg

$$\text{Total alloy} = \frac{55}{2} + \frac{11}{4} = \frac{110+11}{4} = \frac{121}{4} \text{ kg}$$

$$\text{Now Ratio between tin and alloy} = \frac{11}{4} \text{ kg} : \frac{121}{4} \text{ kg}$$

$$= 11 : 121 = 1 : 11$$

Question 2.
Find the compounded ratio of:
(i) 2 : 3 and 4 : 9
(ii) 4 : 5, 5 : 7 and 9 : 11
(iii) $(a - b) : (a + b)$, $(a + b)^2 : (a^2 + b^2)$ and $(a^4 - b^4) : (a^2 - b^2)^2$

Solution:

(i) 2 : 3 and 4 : 9

Compound ratio = $\frac{2}{3} \times \frac{4}{9}$

= $\frac{8}{27}$ or 8 : 27

(ii) 4 : 5, 5 : 7 and 9 : 11

Compound ratio = $\frac{4}{5} \times \frac{5}{7} \times \frac{9}{11} = \frac{36}{77}$

or 36 : 77

(iii) $(a - b) : (a + b), (a + b)^2 : (a^2 + b^2)$

and $(a^4 - b^4) : (a^2 - b^4)^2$

Compound ratio

$= \frac{a-b}{a+b} \times \frac{(a+b)^2}{a^2+b^2} \times \frac{a^4-b^4}{(a^2-b^2)^2}$

$= \frac{a-b}{a+b} \times \frac{(a+b)(a+b)}{a^2+b^2}$

$\times \frac{(a^2+b^2)(a+b)(a-b)}{(a+b)^2(a-b)^2}$

$= \frac{1}{1}$ or 1 : 1

Question 3.
Find the duplicate ratio of
(i) 2 : 3
(ii) √5 : 7
(iii) 5a : 6b
Solution:

(i) Duplicate ratio of 2 : 3 = $(2)^2 : (3)^2$ = 4 : 9

(ii) Duplicate ratio of √5 : 7 = $(\sqrt{5})^2 : (7)^2$ = 5 : 49

(iii) Duplicate ratio of 5a : 6b = $(5a)^2 : (6b)^2$ = $25a^2 : 36b^2$

Question 4.
Find the triplicate ratio of
(i) 3 : 4
(ii) 12 : 13

(iii) $1^3 : 2^3$
Solution:

(i) Triplicate ratio of $3 : 4$

$= (3)^3 : (4)^3$

$= 27 : 64$

(ii) Triplicate ratio of $\dfrac{1}{2} : \dfrac{1}{3} = \left(\dfrac{1}{2}\right)^3 : \left(\dfrac{1}{3}\right)^3$

$= \dfrac{1}{8} : \dfrac{1}{27} = 27 : 8$

(iii) Triplicate ratio of $1^3 : 2^3 = (1^3)^3 : (2^3)^3$

$= (1)^3 : (8)^3 = 1 : 512$

Question 5.
Find the sub-duplicate ratio of
(i) 9 : 16
(ii) 14 : 19,
(iii) $9a^2 : 49b^2$
Solution:

(i) Sub-duplicate ratio of $9 : 16$

$= \sqrt{9} : \sqrt{16}$

$= 3 : 4$

$= \sqrt{9} : \sqrt{16} = 3 : 4$

(ii) Sub-duplicate ratio of $\dfrac{1}{4} : \dfrac{1}{9} = \sqrt{\dfrac{1}{4}} : \sqrt{\dfrac{1}{9}}$

$= \dfrac{1}{2} : \dfrac{1}{3} = 3 : 2$

(iii) Sub-duplicate ratio of $9a^2 : 49b^2$

$= \sqrt{9a^2} : \sqrt{49b^2} = 3a : 7b$

Question 6.
Find the sub-triplicate ratio of
(i) 1 : 216
(ii) 18 : 1125
(iii) $27a^3 : 64b^3$

Solution:

(i) Sub-triplicate ratio of 1 : 216

$= \sqrt[3]{1} : \sqrt[3]{216}$

$= (1^3)^{\frac{1}{3}} : (6^3)^{\frac{1}{3}} = 1 : 6$

(ii) Sub-triplicate ratio of $\dfrac{1}{8} : \dfrac{1}{125}$

$= \left(\dfrac{1}{8}\right)^{\frac{1}{3}} : \left(\dfrac{1}{125}\right)^{\frac{1}{3}} = \left[\left(\dfrac{1}{2}\right)^3\right]^{\frac{1}{3}} : \left[\left(\dfrac{1}{5}\right)^3\right]^{\frac{1}{3}}$

$= \dfrac{1}{2} : \dfrac{1}{5} = 5 : 2$

(iii) Sub-triplicate ratio of $27a^3 : 64b^3$

$= \left[(3a)^3\right]^{\frac{1}{3}} : \left[(4b)^3\right]^{\frac{1}{3}} = 3a : 4b$

Question 7.
Find the reciprocal ratio of
(i) 4 : 7
(ii) $3^2 : 4^2$
(iii) $\frac{1}{9} : 2$

Solution:

(i) Reciprocal ratio of 4 : 7 = 7 : 4

(ii) Reciprocal ratio of $3^2 : 4^2 = 4^2 : 3^2 = 16 : 9$

(iii) Reciprocal ratio of $\frac{1}{9} : 2 = 2 : \frac{1}{9} = 18 : 1$

Question 8.
Arrange the following ratios in ascending order of magnitude:
2 : 3, 17 : 21, 11 : 14 and 5 : 7

Solution:

Writing the given ratios in fraction

$$\frac{2}{3}, \frac{17}{21}, \frac{11}{14}, \frac{5}{7}$$

LCM of 3, 21, 14, 7 = 42

Converting the given ratio as equivalent

$$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42} \; ; \; \frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$$

$$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{42} \; ; \; \frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$$

From above, writing in ascending order,

$$\frac{28}{42}, \frac{30}{42}, \frac{33}{42}, \frac{34}{42} \quad \text{or} \quad \frac{2}{3}, \frac{5}{7}, \frac{11}{14}, \frac{17}{21}$$

or $2 : 3 \; ; \; 5 : 7 \; ; \; 11 : 14$ and $17 : 21$

Question 9.
(i) If A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 : 7, find A : D
(ii) If x : y = 2 : 3, and y : z = 4 : 7, find x : y : z
Solution:

Let A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 : 7

$$\frac{A}{B} = \frac{2}{3}, \frac{B}{C} = \frac{4}{5}, \frac{C}{D} = \frac{6}{7}$$

Multiplying $\dfrac{A}{B} \times \dfrac{B}{C} \times \dfrac{C}{D} = \dfrac{2}{3} \times \dfrac{4}{5} \times \dfrac{6}{7}$

$\therefore \dfrac{A}{D} = \dfrac{16}{35} \Rightarrow A : D = 16 : 35$

(ii) LCM of y's terms 3 and 4 = 12
Making equals of y as 12

$$\frac{x}{y} = \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \text{ or } 8 : 12$$

$$\frac{y}{z} = \frac{4}{7} \times \frac{3}{3} = \frac{12}{21} \text{ or } 12 : 21$$

Then $x : y : z = 8 : 12 : 21$

Question 10.
(i) If A : B = 14:15 and B : C = 17:16, find A : B : C.

(ii) If 3A = 4B = 6C, find A : B : C
Solution:

$A : B = \frac{1}{4} \times \frac{5}{1} = \frac{5}{4}$

$B : C = \frac{1}{7} \times \frac{6}{1} = \frac{6}{7}$

LCM of B's terms 4 and 6 = 12
Making terms of B's; as 12

$\frac{A}{B} = \frac{5 \times 3}{4 \times 3} = \frac{15}{12} = 15 : 12$

$\frac{B}{C} = \frac{6 \times 2}{7 \times 2} = \frac{12}{14} = 12 : 14$

∴ A : B : C = 15 : 12 : 14

(ii) 3A = 4B ⟹ $\frac{A}{B} = \frac{4}{3}$ or A : B = 4 : 3

and 4B = 6C

⟹ $\frac{B}{C} = \frac{6}{4} = \frac{3}{2}$ or B : C = 3 : 2

∴ A : B : C = 4 : 3 : 2

Question 11.
(i) If $3x+5y3x-5y=73$, Find x : y
(ii)) If a : b = 3 : 11, find (15a – 3b) : (9a + 5b). a

Solution:

(i) $\dfrac{3x+5y}{3x-5y} = \dfrac{7}{3}$

$\Rightarrow 9x + 15y = 21x - 35y$ [By cross multiplication]

$\Rightarrow 21x - 9x = 15y + 35y$

$\Rightarrow 12x = 50y \quad \Rightarrow \quad \dfrac{x}{y} = \dfrac{50}{12} = \dfrac{25}{6}$

Hence, $x : y = 25 : 6$

(ii) $a : b = 3 : 11$ or $\dfrac{a}{b} = \dfrac{3}{11}$

Now $\dfrac{15a - 3b}{9a + 5b} = \dfrac{\dfrac{15a}{b} - \dfrac{3b}{b}}{\dfrac{9a}{b} + \dfrac{5b}{b}}$ (Dividing by b)

$= \dfrac{\dfrac{15a}{b} - 3}{\dfrac{9a}{b} + 5} = \dfrac{15 \times \dfrac{3}{11} - 3}{9 \times \dfrac{3}{11} + 5}$

$\left(\text{Substituting the value of } \dfrac{a}{b}\right)$

$= \dfrac{\dfrac{45}{11} - 3}{\dfrac{27}{11} + 5} = \dfrac{\dfrac{45 - 33}{11}}{\dfrac{27 + 55}{11}} = \dfrac{\dfrac{12}{11}}{\dfrac{82}{11}}$

$= \dfrac{12}{11} \times \dfrac{11}{82} = \dfrac{12}{82} = \dfrac{6}{41}$

$\therefore \quad (15a - 3b) : (9a + 5b) = 6 : 41$

Question 12.
(i) If $(4x^2 + xy) : (3xy - y^2) = 12 : 5$, find $(x + 2y) : (2x + y)$.
(ii) If $y (3x - y) : x (4x + y) = 5 : 12$. Find $(x^2 + y^2) : (x + y)^2$.
Solution:

$(4x^2 + xy) : (3xy - y^2) = 12 : 5$

$\Rightarrow \frac{4x^2+xy}{3xy-y^2} = \frac{12}{5}$

$\Rightarrow 20x^2 + 5xy = 36xy - 12y^2$

$\Rightarrow 20x^2 + 5xy - 36xy + 12y^2 = 0$

$\Rightarrow \qquad 20x^2 - 31xy + 12y^2 = 0$

$\Rightarrow \qquad \dfrac{20x^2}{y^2} - \dfrac{31xy}{y^2} + \dfrac{12y^2}{y^2} = 0$

(Dividing by y^2)

$\Rightarrow \qquad 20\left(\dfrac{x}{y}\right)^2 - 31\left(\dfrac{x}{y}\right) + 12 = 0$

$\Rightarrow 20\left(\dfrac{x}{y}\right)^2 - 15\left(\dfrac{x}{y}\right) - 16\left(\dfrac{x}{y}\right) + 12 = 0$

$\Rightarrow 5\left(\dfrac{x}{y}\right)\left[4\left(\dfrac{x}{y}\right) - 3\right] - 4\left[4\left(\dfrac{x}{y}\right) - 3\right] = 0$

$\Rightarrow \left[4\left(\dfrac{x}{y}\right) - 3\right]\left[5\left(\dfrac{x}{y}\right) - 4\right] = 0$

Either $4\left(\dfrac{x}{y}\right) - 3 = 0$, then $4\left(\dfrac{x}{y}\right) = 3$

$\Rightarrow \qquad \dfrac{x}{y} = \dfrac{3}{4}$

or $\quad 5\left(\dfrac{x}{y}\right) - 4 = 0,$

then $\quad 5\left(\dfrac{x}{y}\right) = 4 \qquad \Rightarrow \quad \dfrac{x}{y} = \dfrac{4}{5}$

Now $\dfrac{x+2y}{2x+y} = \dfrac{\dfrac{x}{y}+2}{2\dfrac{x}{y}+1}$ (Dividing by y)

(a) When $\dfrac{x}{y} = \dfrac{3}{4}$, then

$= \dfrac{\dfrac{x}{y}+2}{2\dfrac{x}{y}+1} = \dfrac{\dfrac{3}{4}+2}{2\times\dfrac{3}{4}+1} = \dfrac{\dfrac{11}{4}}{\dfrac{3}{2}+1} = \dfrac{\dfrac{11}{4}}{\dfrac{5}{2}}$

$= \dfrac{11}{4} \times \dfrac{2}{5} = \dfrac{11}{10}$

$\therefore (x+2y) : (2x+y) = 11 : 10$

(b) When $\dfrac{x}{y} = \dfrac{4}{5}$, then

$$\dfrac{x + 2y}{2x + y} = \dfrac{\dfrac{x}{y} + 2}{2\dfrac{x}{y} + 1} = \dfrac{\dfrac{4}{5} + 2}{2 \times \dfrac{4}{5} + 1} = \dfrac{\dfrac{14}{5}}{\dfrac{8}{5} + 1}$$

(Dividing by y)

$$= \dfrac{\dfrac{14}{5}}{\dfrac{13}{5}} = \dfrac{14}{5} \times \dfrac{5}{13} = \dfrac{14}{13}$$

Hence $\dfrac{x + 2y}{2x + y} = \dfrac{11}{10}$ or $\dfrac{14}{13}$

$\therefore \qquad \dfrac{x + 2y}{2x + y} = 11 : 10 \text{ or } 14 : 13$

(ii) If $y(3x - y) : x(4x + y) = 5 : 12$

Find $(x^2 + y^2) : (x + y)^2$

$$\dfrac{3xy - y^2}{4x^2 + xy} = \dfrac{5}{12}$$

$\Rightarrow 36xy - 12y^2 = 20x^2 + 5xy$

$\Rightarrow 20x^2 + 5xy - 36xy + 12y^2 = 0$

$\Rightarrow \qquad 20x^2 - 31xy + 12y^2 = 0$

$\Rightarrow \quad 20\dfrac{x^2}{y^2} - 31\dfrac{xy}{y^2} + \dfrac{12y^2}{y^2} = 0$

(Dividing by y^2)

$\Rightarrow 20\left(\dfrac{x^2}{y^2}\right) - 31\left(\dfrac{xy}{y^2}\right) + 12 = 0$

$\Rightarrow 20\left(\dfrac{x}{y}\right)^2 - 15\left(\dfrac{x}{y}\right) - 16\left(\dfrac{x}{y}\right) + 12 = 0$

$\Rightarrow 5\left(\dfrac{x}{y}\right)\left[4\left(\dfrac{x}{y}\right) - 3\right] - 4\left[4\left(\dfrac{x}{y}\right) - 3\right] = 0$

$\Rightarrow \left[4\left(\dfrac{x}{4}\right) - 3\right]\left[5\left(\dfrac{x}{y}\right) - 4\right] = 0$

Either $\left[4\left(\dfrac{x}{y}\right) - 3\right] = 0$,

then $4\left(\dfrac{x}{y}\right) = 3 \quad\Rightarrow\quad \dfrac{x}{y} = \dfrac{3}{4}$

or $\left[5\left(\dfrac{x}{y}\right) - 4\right] = 0$

then $5\left(\dfrac{x}{y}\right) = 4 \quad\Rightarrow\quad \dfrac{x}{y} = \dfrac{4}{5}$

(a) when $\quad \dfrac{x}{y} = \dfrac{3}{4}$

then $(x^2 + y^2) : (x + y)^2$

$= \dfrac{x^2 + y^2}{(x + y)^2} = \dfrac{\dfrac{x^2}{y^2} + \dfrac{y^2}{y^2}}{\dfrac{1}{y^2}(x + y)^2}$ (Dividing by y^2)

$= \dfrac{\dfrac{x^2}{y^2} + 1}{\left(\dfrac{x}{y} + 1\right)^2}$

$$= \frac{\left(\frac{3}{4}\right)^2 + 1}{\left(\frac{3}{4} + 1\right)^2} = \frac{\frac{9}{16} + 1}{\left(\frac{7}{4}\right)^2}$$

$$= \frac{\frac{25}{16}}{\frac{49}{16}} = \frac{25}{16} \times \frac{16}{49} = \frac{25}{49}$$

$\therefore (x^2 + y^2) : (x + y)^2 = 25 : 49$

(b) When $\frac{x}{y} = \frac{4}{5}$, then

$$\frac{x^2 + y^2}{(x + y)^2} = \frac{\frac{x^2}{y^2} + 1}{\left(\frac{x}{y} + 1\right)^2} = \frac{\left(\frac{x}{y}\right)^2 + 1}{\left(\frac{x}{y} + 1\right)^2}$$

$$= \frac{\left(\frac{4}{5}\right)^2 + 1}{\left(\frac{4}{5} + 1\right)^2} = \frac{\frac{16}{25} + 1}{\left(\frac{9}{5}\right)^2} = \frac{\frac{41}{25}}{\frac{81}{25}} = \frac{41}{25} \times \frac{25}{81} = \frac{41}{81}$$

$\therefore (x^2 + y^2) : (x + y)^2 = 41 : 81$

Question 13.
(i) If (x – 9) : (3x + 6) is the duplicate ratio of 4 : 9, find the value of x.
(ii) If (3x + 1) : (5x + 3) is the triplicate ratio of 3 : 4, find the value of x.
(iii) If (x + 2y) : (2x – y) is equal to the duplicate ratio of 3 : 2, find x : y.
Solution:

(i) $\frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$

$\Rightarrow \frac{x-9}{3x+6} = \frac{16}{81}$

$\Rightarrow 81x - 729 = 48x + 96$

$\Rightarrow 81x - 48x = 96 + 729$

$\Rightarrow 33x = 825 \Rightarrow x = \dfrac{825}{33} = 25$

(ii) If $(3x + 1) : (5x + 3)$ is the triplicate ratio of $3 : 4$,

then $\dfrac{3x+1}{5x+3} = \dfrac{(3)^3}{(4)^3} = \dfrac{27}{64}$

$\Rightarrow 64(3x + 1) = 27(5x + 3)$

$\Rightarrow 192x + 64 = 135x + 81$

$\Rightarrow 192x - 135x = 81 - 64$

$\Rightarrow 57x = 17 \Rightarrow x = \dfrac{17}{57}$

Hence $x = \dfrac{17}{57}$

(iii) If $(x + 2y) : (2x - y)$ is the duplicate ratio of $3 : 2$,

then $\dfrac{x+2y}{2x-y} = \dfrac{(3)^2}{(2)^2} = \dfrac{9}{4}$

$\Rightarrow 9(2x - y) = 4(x + 2y)$

$\Rightarrow 18x - 9y = 4x + 8y$

$\Rightarrow 18x - 4x = 8y + 9y$

$\Rightarrow 14x - 17y \Rightarrow \dfrac{x}{y} = \dfrac{17}{14}.$

$\therefore \quad x : y = 17 : 14$

Question 14.
(i) Find two numbers in the ratio of 8 : 7 such that when each is decreased by 12₁₂, they are in the ratio 11 : 9.

(ii) The income of a man is increased in the ratio of 10 : 11. If the increase in his income is Rs 600 per month, find his new income.

Solution:

(i) The ratio is 8 : 7

Let the numbers be 8x and 7x,

According to condition,

$$\frac{8x - \dfrac{25}{2}}{7x - \dfrac{25}{2}} = \frac{11}{9} \Rightarrow \frac{\dfrac{16x - 25}{2}}{\dfrac{14x - 25}{2}} = \frac{11}{9}$$

$$\Rightarrow \frac{(16x - 25) \times 2}{2(14x - 25)} = \frac{11}{9}$$

$$\Rightarrow \frac{16x - 25}{14x - 25} = \frac{11}{9}$$

$$\Rightarrow 154x - 275 = 144x - 225$$

$$\Rightarrow 154x - 144x = 275 - 225$$

$$\Rightarrow 10x = 50$$

$$\therefore x = \frac{50}{10} = 5$$

$\therefore$ Numbers are $8x = 8 \times 5 = 40$

and $7x = 7 \times 5 = 35$

(ii) Let the present income $= 10x$

then increased income $= 11x$

$\therefore$ Increase per month $= 11x - 10x = x$

$\therefore x = ₹600$

Now his new income $= 11x = 11 \times 600$

$= ₹6600$

Question 15.

(i) A woman reduces her weight in the ratio 7 : 5. What does her weight become if originally it was 91 kg?

(ii) A school collected Rs 2100 for charity. It was decided to divide the money between an orphanage and a blind school in the ratio of 3 : 4. How much money did each receive?

Solution:

(i) Ratio between the original weight and reduced weight = 7 : 5

Let original weight = 7x

then reduced weight = 5x

If original weight = 91 kg.

then reduced weight $= \dfrac{91 \times 5x}{7x} = 65$ kg.

(ii) Total amount to be distributed = ₹2100

Ratio between orphanage and a blind school
= 3 : 4

Sum of ratios = 3 + 4 = 7

∴ Orphanage school's share $= ₹2100 \times \dfrac{3}{7}$

= ₹900

Blind School's share $= ₹2100 \times \dfrac{4}{7} = ₹1200$

Question 16.
(i) The sides of a triangle are in the ratio 7 : 5 : 3 and its perimeter is 30 cm.
Find the lengths of sides.
(ii) If the angles of a triangle are in the ratio 2 : 3 : 4, find the angles.

Solution:

(i) Perimeter of a triangle = 30 cm.

Ratio among sides = 7 : 5 : 3

Sum of ratios 7 + 5 + 3 = 15

Length of first side $= 30 \times \dfrac{7}{15} = 14 cm$

Length of second side $= 30 \times \dfrac{5}{15} = 10 cm$

Length of third side $= 30 \times \dfrac{3}{15} = 6$ cm.

∴ Sides are 14cm, 10cm, 6 cm

(ii) Sum of angles of a triangle $= 180°$

Ratio among angles $= 2 : 3 : 4$

Sum of ratios $= 2 + 3 + 4 = 9$

∴ First angle $= 180° \times \dfrac{2}{9} = 40°$

Second angle $= 180° \times \dfrac{3}{9} = 60°$

Third angle $= 180° \times \dfrac{4}{9} = 80°$

∴ Angles are 40°, 60° and 80°

Question 17.

Three numbers are in the ratio $1/2 : 1/3 : 1/4$ If the sum of their squares is 244, find the numbers.

Solution:

The ratio of three numbers $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$= \frac{6:4:3}{12}$

$= 6 : 4 : 3$

Let first number 6x, second 4x and third 3x

∴ According to the condition

$(6x)^2 + (4x)^2 + (3x)^2 = 244$

$\Rightarrow 36x^2 + 16x^2 + 9x^2 = 244$

$\Rightarrow \quad 61 x^2 = 244$

$\Rightarrow x^2 = \dfrac{244}{61} = 4 = (2)^2$

∴ $\quad x = 2$

∴ first number $= 6x = 6 \times 2 = 12$

second number $= 4x = 4 \times 2 = 8$

and third number $= 3x = 3 \times 2 = 6$

Question 18.
(i) A certain sum was divided among A, B and C in the ratio 7 : 5 : 4. If B got Rs 500 more than C, find the total sum divided.
(ii) In a business, A invests Rs 50000 for 6 months, B Rs 60000 for 4 months and C, Rs 80000 for 5 months. If they together earn Rs 18800 find the share of each.
Solution:

(i) Ratio between A, B and C = 7 : 5 : 4

Let A's share = 7x

B's share = 5x

and C's share = 4x

Total sum = 7x + 5x + 4x = 16x

Now according to the condition,

$5x - 4x = 500 \Rightarrow x = 500$

∴ Total sum = 16x = 16 × 500 = ₹8000

(ii) A's 6 months investment = ₹50000

∴ A's 1 month investment

= ₹50000 × 6 = ₹300000

B's 4 month's investment = ₹60000

∴ B's 1 month investment

= Rs. 60000 × 4 = ₹240000

C's 5 months investment = ₹80000

∴ C's 1 month investment

= ₹80000 × 5 = ₹400000

∴ Ratio between their investments

= 300000 : 240000 : 400000

= 30 : 24 : 40

Sum of ratios = 30 + 24 + 40 = 94

Total earnings = ₹18800

∴ A's share = $\dfrac{30}{94} \times 18800$ = ₹6000

B's share = $\dfrac{24}{94} \times 18800$ = ₹4800

C's share = $\dfrac{40}{94} \times 18800$ = ₹8000

Question 19.
(i) In a mixture of 45 liters, the ratio of milk to water is 13 : 2. How much water

must be added to this mixture to make the ratio of milk to water as 3 : 1?
(ii) The ratio of the number of boys to the number of girls in a school of 560 pupils is 5 : 3. If 10 new boys are admitted, find how many new girls may be admitted so that the ratio of the number of boys to the number of girls may change to 3 : 2.
Solution:

(i) Mixture of milk and water = 45 litres

Ratio of milk and water =13 : 2

Sum of ratio = 13 + 2 = 15

$\therefore$ Quantity of milk $= \dfrac{45 \times 13}{15} = 39$ litres

and quantity of water $= 45 \times \dfrac{2}{15} = 6$ litres

Let x litre of water be added, then water
$$= (6 + x)\ \text{litres}$$

Now new ratio = 3 : 1

$\therefore$ 39 : (6 + x) = 3 : 1

$\dfrac{39}{6+x} = \dfrac{3}{1} \Rightarrow 39 = 18 + 3x$

$\Rightarrow \quad 3x = 39 - 18 = 21$

$\therefore x = \dfrac{21}{3} = 7$ litres.

$\therefore$ 7 litres of water is to be added.

(ii) Ratio between boys and girls = 5 : 3

No. of pupils = 560

Sum of ratios = 5 + 3 = 8

$\therefore$ No. of boys = $\dfrac{5}{8} \times 560 = 350$

and no. of girls = $\dfrac{3}{8} \times 560 = 210$

No. of new boys admitted = 10

$\therefore$ Total number of boys = 350 + 10 = 360

Let the no. of girls admitted = x

$\therefore$ Total number of girls = 210 + x

Now according to the condition,

$360 : 210 + x = 3 : 2 \Rightarrow \dfrac{360}{210+x} = \dfrac{3}{2}$

$\Rightarrow \quad 630 + 3x = 720$

$\Rightarrow \quad 3x = 720 - 630 = 90$

$\therefore x = \dfrac{90}{3} = 30$

$\therefore$ No of girls to be admitted = 30

Question 20.
(i) The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves Rs 80 every month, find their monthly pocket money.
(ii) In class X of a school, the ratio of the number of boys to that of the girls is 4 : 3. If there were 20 more boys and 12 fewer girls, then the ratio would have been 2 : 1, How many students were there in the class?
Solution:

(i) Let the monthly pocket money of Ravi and Sanjeev be 5x and 7x respectively.

Also, let their expenditure be 3y and 5y respectively.

So, $5x - 3y = 80$ ...(i)

and $7x - 5y = 80$ ...(ii)

Multiplying (i) by 7 and (ii) by 5 and, subtracting, we get

$35x - 21y = 560$

$35x - 25y = 400$

$- + -$
_____

$ 4y = 160 \Rightarrow y = 40$

From (i), $5x = 80 + 3 \times 40 = 200 \Rightarrow x = 40$

So, monthly pocket money of Ravi

$= ₹5 \times 40 = ₹200$

(*ii*) Let the number of students in the class
$= x$

Ratio of boys and girls $= 4 : 3$

$\therefore$ No. of boys $= \dfrac{4x}{7}$

and no. of girls $= \dfrac{3x}{7}$

According to the problem,

$\left(\dfrac{4x}{7} + 20\right) : \left(\dfrac{3x}{7} - 12\right) = 2 : 1$

$\dfrac{4x + 140}{7} : \dfrac{3x - 84}{7} : 2 : 1$

$\Rightarrow \dfrac{4x + 140}{7} \times \dfrac{7}{3x - 84} = \dfrac{2}{1}$

$\Rightarrow \dfrac{4x + 140}{3x - 84} = \dfrac{2}{1}$

$\Rightarrow 6x - 168 = 4x + 140$

$\Rightarrow \quad 6x - 4x = 140 + 168$

$\Rightarrow 2x = 308 \Rightarrow \quad x = \dfrac{308}{2} = 154$

Hence, number of students $= 154$

Question 21.
In an examination, the ratio of passes to failures was 4 : 1. If 30 less had appeared and 20 less passed, the ratio of passes to failures would have been 5 : 1. How many students appeared for the examination

Solution:

Let the number of passes = 4x

and number of failures = x

The total number of students appeared = 4x + x = 5x

In the second case, the number of students appeared = 5x − 30

and number of passes = 4x − 20

∴ No. of failures = (5x − 30) − (4x − 20)

= 5x − 30 − 4x + 20 = x − 10

According to the condition

$$\frac{4x - 20}{x - 10} = \frac{5}{1}$$

⇒ 5x − 50 = 4x − 20

⇒ 5x − 4x = − 20 + 50

⇒ x = 30

∴ Number of students appeared = 5x

= 5 × 30 = 150