

Question 1.

Find the product of:

- (i) $4x^3$ and $-3xy$
- (ii) $2xyz$ and 0
- (iii) $-\frac{2}{3}p^2q$, $\frac{3}{4}pq^2$ and $5pqr$
- (iv) $-7ab$, $-3a^3$ and $-\frac{2}{7}ab^2$
- (v) $-\frac{1}{2}x^2$, $-\frac{3}{5}xy$, $\frac{2}{3}yz$ and $\frac{5}{7}xyz$

Solution:

Product of

$$(i) 4x^3 \text{ and } -3xy = 4x^3 \times (-3xy) = -12x^{3+1}y = -12x^4y$$

$$(ii) 2xyz \text{ and } 0 = 2xyz \times 0 = 0$$

$$(iii) \left(-\frac{2}{3}p^2q\right) \times \left(\frac{3}{4}pq^2\right) \times (5pqr)$$

$$= -\frac{2}{3} \times \frac{3}{4} \times 5 \times p^2q \times pq^2 \times pqr$$

$$= -\frac{5}{2} p^4 q^4 r$$

$$(iv) (-7ab) \times (-3a^3) \times \left(-\frac{2}{7}ab^2\right)$$

$$= (-7) \times (-3) \times \left(-\frac{2}{7}\right) \times ab \times a^3 \times ab^2$$

$$= -6a^5b^3.$$

$$(v) \left(-\frac{1}{2}x^2\right) \times \left(-\frac{3}{5}xy\right) \times \left(\frac{2}{3}yz\right) \times \left(\frac{5}{7}xyz\right)$$

$$= \left(-\frac{1}{2}\right) \times \left(-\frac{3}{5}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{5}{7}\right) \\ \times x^2 \times xy \times yz + xyz$$

$$= \frac{1}{7} x^4 y^3 z^2$$

Question 2.

Multiply:

$$(i) (3x - 5y + 7z) \text{ by } -3xyz$$

$$(ii) (2p^2 - 3pq + 5q^2 + 5) \text{ by } -2pq$$

$$(iii) \left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right) \text{ by } 35ab$$

$$(iv) (4x^2 - 10xy + 7y^2 - 8x + 4y + 3) \text{ by } 3xy$$

Solution:

$$(i) -3xyz (3x - 5y + 7z)$$

$$= (-3xyz) \times 3x + (-3xyz) \times (-5y) + (-3xyz) \times (7z)$$

$$= -9x^2yz + 15xyz^2 - 21xyz^2$$

$$\begin{aligned} \text{(ii)} \quad & -2pq(2p^2 - 3pq + 5q^2 + 5) \\ &= (-2pq) \times 2p^2 + (-2pq) \times (-3pq) + (-2pq) \times (5q^2) + \\ &\quad (-2pq) \times 5 \\ &= -4p^3q + 6p^2q^2 - 10pq^3 - 10pq \end{aligned}$$

(iii) $\left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right)$ by $35ab$

$$= \frac{2}{3}a^2b \times 35ab - \frac{4}{5}ab^2 \times 35ab = \frac{2}{7}ab \times 35ab + 3 \times 35ab$$
$$= \frac{70}{3}a^3b^2 - 28a^2b^3 + 10a^2b^2 + 105ab$$

(iv) $(4x^2 - 10xy + 7y^2 - 8x + 4y + 3)$ by $3xy$

$$4x^2 \times 3xy - 10xy \times 3xy + 7y^2 \times 3xy - 8x \times 3xy + 4y \times 3xy + 3 \times 3xy$$
$$= 12x^3y - 30x^2y^2 + 21xy^3 - 24x^2y + 12xy^2 + 9xy$$

Question 3.

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:

(i) (p^2q, pq^2)

(ii) $(5xy, 7xy^2)$

Solution:

(i) Sides of a rectangle are p^2q and pq^2

$$\text{Area} = p^2q \times pq^2 = p^{2+1}q^{2+1} = p^3q^3$$

(ii) Sides are $5xy$ and $7xy^2$

$$\text{Area} = 5xy \times 7xy^2 = 35x^{1+1} \times y^{1+2} = 35x^2y^3$$

Question 4.

Find the volume of rectangular boxes with the following length, breadth and height respectively:

(i) $5ab, 3a^2b, 7a^4b^2$

(ii) $2pq, 4q^2, 8rp$

Solution:

Length, breadth and height of a rectangular box are

(i) $5ab, 3a^2b, 7a^4b^2$

$$\therefore \text{Volume} = \text{Length} \times \text{breadth} \times \text{height}$$

$$= 5ab \times 3a^2b \times 7a^4b^2$$

$$= 5 \times 3 \times 7 \times a^{1+2+4} \times b^{1+1+2}$$

$$= 105a^7b^4$$

(ii) $2pq, 4q^2, 8rp$

$$\therefore \text{Volume} = 2pq \times 4q^2 \times 8rp$$

$$= 2 \times 4 \times 8 \times p^{1+1} \times q^{1+2} \times r$$

$$= 64p^2q^3r$$

Question 5.

Simplify the following expressions and evaluate them as directed:

(i) $x^2(3 - 2x + x^2)$ for $x = 1; x = -1; x = \frac{2}{3}$ and $x = -\frac{1}{2}$

(ii) $5xy(3x + 4y - 7) - 3y(xy - x^2 + 9) - 8$ for $x = 2, y = -1$

Solution:

(i) $x^2(3 - 2x + x^2)$

for $x = 1; x = -1; x = -1; x = \frac{2}{3}$ and $x = -\frac{1}{2}$

$$x^2(3 - 2x + x^2) = 3x^2 - 2x^3 + x^4$$

(a) $x = 1$, then

$$\begin{aligned}3x^2 - 2x^3 + x^4 &= 3(1)^2 - 2(1)^3 + (1)^4 \\&= 3 \times 1 - 2 \times 1 + 1 \\&= 3 - 2 + 1 = 2\end{aligned}$$

(b) $x = -1$

$$\begin{aligned}3x^2 - 2x^3 + x^4 &= 3(-1)^2 - 2(-1)^3 + (-1)^4 \\&= 3 \times 1 - 2 \times (-1) + 1 = 3 + 2 + 1 = 6\end{aligned}$$

(c) $x = \frac{2}{3}$

$$3x^2 - 2x^3 + x^4 = 3\left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4$$

$$= 3 \times \frac{4}{9} - 2 \times \frac{8}{27} + \frac{16}{81}$$

$$= \frac{4}{3} - \frac{16}{27} + \frac{16}{81}$$

$$= \frac{108 - 48 + 16}{81} = \frac{124 - 48}{81} = \frac{76}{81}$$

$$(d) x = -\frac{1}{2}$$

$$3x^2 - 2x^3 + x^4$$

$$= 3\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^4$$

$$= 3\left(\frac{1}{4}\right) - 2\left(-\frac{1}{8}\right) + \frac{1}{16}$$

$$= \frac{3}{4} + \frac{1}{4} + \frac{1}{16}$$

$$= \frac{12+4+1}{16} = \frac{17}{16}$$

$$(ii) 5xy(3x + 4y - 7) - 3y(xy - x^2 + 9) - 8$$

$$= 15x^2y + 20xy^2 - 35xy - 3xy^2 + 3x^2y - 21y - 8$$

$$= 18x^2y + 17xy^2 - 35xy - 27y - 8$$

When $x = 2, y = -1$

$$= 18(2)^2 \times (-1) + 17(2)(-1)^2 - 35(2)(-1) - 27(-1) - 8$$

$$= 18 \times 4 \times (-1) + 17 \times 2 \times 1 - 35 \times 2 \times (-1) - 27 \times (-1)$$

$$- 8$$

$$= -74 + 34 + 70 + 27 - 8$$

$$= 131 - 80 = 51$$

Question 6.

Add the following:

(i) $4p(2 - p^2)$ and $8p^3 - 3p$

(ii) $7xy(8x + 2y - 3)$ and $4xy^2(3y - 7x + 8)$

Solution:

Add

(i) $4p(2 - p^2)$ and $8p^3 - 3p$

$$= 8p - 4p^3 + 8p^3 - 3p$$

$$= 5p + 4p^3$$

$$= 4p^3 + 5p$$

(ii) $7xy(8x + 2y - 3)$ and $4xy^2(3y - 7x + 8)$

$$= 56x^2y + 14xy^2 - 21xy + 12xy^3 - 28x^2y^2 + 32xy^2$$

$$= 12xy^3 - 28x^2y^2 + 56x^2y + 46xy^2 = 21xy$$

Question 7.

Subtract:

(i) $6x(x - y + z) - 3y(x + y - z)$ from $2z(-x + y + z)$

(ii) $7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$ from

$$3y(4x^2y - 5xy + 8xy^2)$$

Solution:

(i) $6x(x - y + z) - 3y(x + y - z)$ from $2z(-x + y + z)$

$$6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz$$
 from $-2xz + 2yz + 2z^2$

$$= (-2xz + 2yz + 2z^2) - (6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz)$$

$$= -2xz + 2yz + 2z^2 - 6x^2 + 6xy - 6xz + 3xy + 3y^2 - 3yz$$

$$= 9xy - yz - 8zx - 6x^2 + 3y^2 + 2z^2$$

$$(ii) 7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$$

$$3y(4x^2y - 5xy + 8xy^2)$$

$$7x^3y - 14x^2y^2 + 21xy^3 - 8x^3y + 32x^2y - 56x^2y^2$$

$$12x^2y^2 - 15xy^2 + 24xy^3$$

$$= (12x^2y^2 - 15xy^2 + 24xy^3) - (7x^3y - 14x^2y^2 + 21xy^3 -$$

$$8x^3y + 32x^2y - 56x^2y^2$$

$$= 12x^2y^2 - 15xy^2 + 24xy^3 - 7x^3y + 14x^2y^2 - 12xy^3 +$$

$$8x^3y - 32x^2y + 56x^2y^2$$

$$= 82x^2y^2 + 3xy^3 + x^3y - 15xy^2 - 32x^2y$$