

Question 1.

Multiply:

(i) $(5x - 2)$ by $(3x + 4)$

(ii) $(ax + b)$ by $(cx + d)$

(iii) $(4p - 7)$ by $(2 - 3p)$

(iv) $(2x^2 + 3)$ by $(3x - 5)$

(v) $(1.5a - 2.5b)$ by $(1.5a + 2.5b)$

(vi) $\left(\frac{3}{7}p^2 + 4q^2\right)$ by $7\left(p^2 - \frac{3}{4}q^2\right)$

Solution:

(i) $(5x - 2)$ by $(3x + 4)$

$$= 5x(3x + 4) - 2(3x + 4)$$

$$15x^2 + 20x - 6x - 8$$

$$= 15x^2 + 14x - 8$$

(ii) $(ax + b)$ by $(cx + d)$

$$= ax(cx + d) + b(cx + d)$$

$$= acx^2 + adx + bcx + bd$$

(iii) $(4p - 7)$ by $(2 - 3p)$

$$= (4p - 7)(2 - 3p)$$

$$= 4p(2 - 3p) - 7(2 - 3p)$$

$$= 8p - 12p^2 - 14 + 21p$$

$$= 29p - 12p^2 - 14$$

(iv) $(2x^2 + 3)$ by $(3x - 5)$

$$= (2x^2 + 3)(3x - 5)$$

$$= 2x^2(3x - 5) + 3(3x - 5)$$

$$= 6x^3 - 10x^2 + 9x - 15$$

$$\begin{aligned}
 & \text{(v)} (1.5a - 2.5b) \text{ by } (1.5a + 2.5b) \\
 &= (1.5a - 2.5b)(1.5a + 2.5b) \\
 &= 1.5a(1.5 + 2.5b) - 2.5b(1.5a + 2.5b) \\
 &= 2.25a^2 + 3.75ab - 3.75a^2 - 6.25b^2 \\
 &= 2.25a^2 - 6.25b^2
 \end{aligned}$$

$$\text{(vi)} \left(\frac{3}{7}p^2 + 4q^2 \right) \text{ by } 7\left(p^2 - \frac{3}{4}q^2 \right)$$

$$\left(\frac{3}{7}p^2 + 4q^2 \right) \times 7\left(p^2 - \frac{3}{4}q^2 \right)$$

$$= 7\left(\frac{3}{7}p^2 + 4q^2 \right) \left(p^2 - \frac{3}{4}q^2 \right)$$

$$= 7\left[\frac{3}{7}p^2\left(p^2 - \frac{3}{4}q^2 \right) + 4q^2\left(p^2 - \frac{3}{4}q^2 \right) \right]$$

$$= 7\left[\frac{3}{7}p^4 - \frac{9}{28}p^2q^2 + 4p^2q^2 - 3q^4 \right]$$

$$= 3p^4 - \frac{9}{4}p^2q^2 + 28p^2q^2 - 21q^4$$

$$= 3p^4 - \frac{9p^2q^2 + 112p^2q^2}{4} - 21q^4$$

$$= 3p^4 + \frac{103}{4}p^2q^2 - 21q^4$$

Question 2.

Multiply:

(i) $(x - 2y + 3)$ by $(x + 2y)$

(ii) $(3 - 5x + 2 \times 2)$ by $(4x - 5)$

Solution:

(i) $(x - 2y + 3)$ by $(x + 2y)$

$$= x(x + 2y) - 2y(x + 2y) + 3(x + 2y)$$

$$= x^2 + 2xy - 2xy - 4y^2 + 3x + 6y$$

$$= x^2 - 4y^2 + 3x + 6y$$

$$\begin{aligned}
 & \text{(ii) } (3 - 5x + 2x^2) \text{ by } (4x - 5) \\
 &= (4x - 5)(3 - 5x + 2x^2) \\
 &= 4x(3 - 5x + 2x^2) - 5(3 - 5x + 2x^2) \\
 &= 12x - 20x^2 + 8x^3 - 15 + 25x - 10x^2 \\
 &= 8x^3 - 30x^2 + 37x - 15
 \end{aligned}$$

Question 3.

Multiply:

$$\text{(i) } (3x^2 - 2x - 1) \text{ by } (2x^2 + x - 5)$$

$$\text{(ii) } (2 - 3y - 5y^2) \text{ by } (2y - 1 + 3y^2)$$

Solution:

$$\begin{aligned}
 & \text{(i) } (3x^2 - 2x - 1) \text{ by } (2x^2 + x - 5) \\
 &= (3x^2 - 2x - 1)(2x^2 + x - 5) \\
 &= 3x^2(2x^2 + x - 5) - 2x(2x^2 + x - 5) - 1(2x^2 + x - 5) \\
 &= 6x^4 + 3x^3 - 15x^2 - 4x^3 - 2x^2 + 10x - 2x^2 - x + 5 \\
 &= 6x^4 - x^3 - 19x^2 + 9x + 5
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii) } (2 - 3y - 5y^2) \text{ by } (2y - 1 + 3y^2) \\
 &= 2(2y - 1 + 3y^2) - 3y(2y - 1 + 3y^2) - 5y^2(2y - 1 + 3y^2) \\
 &= 4y - 2 + 6y^2 - 6y^2 + 3y - 9y^3 - 10y^3 + 5y^2 - 15y^4 \\
 &= -15y^4 - 19y^3 + 5y^2 + 7y - 2
 \end{aligned}$$

Question 4.

Simplify:

$$(i) (x^2 + 3)(x - 3) + 9$$

$$(ii) (x + 3)(x - 3)(x + 4)(x - 4)$$

$$(iii) (x + 5)(x + 6)(x + 7)$$

$$(iv) (p + q - 2r)(2p - q + r) - 4qr$$

$$(v) (p + q)(r + s) + (p - q)(r - s) - 2(pr + qs)$$

$$(vi) (x + y + z)(x - y + z) + (x + y - z)(-x + y + z) - 4zx$$

Solution:

$$(i) (x^2 + 3)(x - 3) + 9$$

$$= x^2(x - 3) + 3(x - 3) + 9$$

$$= x^2 - 3x^2 + 3x - 9 + 9$$

$$= x^3 - 3x^2 + 3x$$

$$(ii) (x + 3)(x - 3)(x + 4)(x - 4)$$

$$= \{(x + 3)(x - 3)\} \{(x + 4)(x - 4)\}$$

$$= \{x(x - 3) + 3(x - 3)\} \{x(x - 4) + 4(x - 4)\}$$

$$= (x^2 - 3x + 3x - 9) \{x^2 - 4x + 4x - 16\}$$

$$= (x^2 - 9)(x^2 - 16)$$

$$= x^2(x^2 - 16) - 9(x^2 - 16)$$

$$= x^4 - 16x^2 - 9x^2 + 144$$

$$= x^4 - 25x^2 + 144$$

$$(iii) (x + 5)(x + 6)(x + 7)$$

$$= (x^2 + 6x + 5x + 30)(x + 7)$$

$$= (x^2 + 11x + 30)(x + 7)$$

$$= x(x^2 + 11x + 30) + 7(x^2 + 11x + 30)$$

$$= x^3 + 11x^2 + 30x + 7x^2 + 77x + 210$$

$$= x^3 + 18x^2 + 107x + 210$$

Question 4.

Simplify:

(i) $(x^2 + 3)(x - 3) + 9$

(ii) $(x + 3)(x - 3)(x + 4)(x - 4)$

(iii) $(x + 5)(x + 6)(x + 7)$

(iv) $(p + q - 2r)(2p - q + r) - 4qr$

(v) $(p + q)(r + s) + (p - q)(r - s) - 2(pr + qs)$

(vi) $(x + y + z)(x - y + z) + (x + y - z)(-x + y + z) - 4zx$

Solution:

(i) $(x^2 + 3)(x - 3) + 9$

$$= x^2(x - 3) + 3(x - 3) + 9$$

$$= x^2 - 3x^2 + 3x - 9 + 9$$

$$= x^3 - 3x^2 + 3x$$

(ii) $(x + 3)(x - 3)(x + 4)(x - 4)$

$$= \{(x + 3)(x - 3)\} \{(x + 4)(x - 4)\}$$

$$= \{x(x - 3) + 3(x - 3)\} \{x(x - 4) + 4(x - 4)\}$$

$$= (x^2 - 3x + 3x - 9) \{x^2 - 4x + 4x - 16\}$$

$$= (x^2 - 9)(x^2 - 16)$$

$$= x^2(x^2 - 16) - 9(x^2 - 16)$$

$$= x^4 - 16x^2 - 9x^2 + 144$$

$$= x^4 - 25x^2 + 144$$

(iii) $(x + 5)(x + 6)(x + 7)$

$$= (x^2 + 6x + 5x + 30)(x + 7)$$

$$= (x^2 + 11x + 30)(x + 7)$$

$$= x(x^2 + 11x + 30) + 7(x^2 + 11x + 30)$$

$$= x^3 + 11x^2 + 30x + 7x^2 + 77x + 210$$

$$= x^3 + 18x^2 + 107x + 210$$

$$\begin{aligned}
 & \text{(iv)} \quad (p + q - 2r)(2p - q + r) - 4qr \\
 &= p(2p - q + r) + q(2p - q + r) - 2r(2p - q + r) - 4qr \\
 &= 2p^2 - pq + pr + 2pq - q^2 + qr - 4pr + 2qr - 2r^2 - 4qr \\
 &= 2p^2 - q^2 - 2r^2 + pq - 3pr - 2qr
 \end{aligned}$$

$$\begin{aligned}
 & \text{(v)} \quad (p + q)(r + s) + (p - q)(r - s) - 2(pr + qs) \\
 &= pr + ps + qr + qs + pr - ps - qr + qs - 2pr - 2qs = 0
 \end{aligned}$$

$$\begin{aligned}
 & \text{(vi)} \quad (x + y + z)(x - y + z) + (x + y - z)(-x + y + z) - 4zx \\
 &= x^2 - xy + xz + xy - y^2 + yz + xz - yz + z^2 - x^2 + xy + \\
 & \quad xz \\
 &= -xy + x^2 + yx + xz - yz - z^2 - 4zx = 0
 \end{aligned}$$

Question 5.

If two adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 - 2xy + 3y^2$, find its area.

Solution:

Adjacents sides of a rectangle are

$$5x^2 + 25xy + 4y^2 \text{ and}$$

$$2x^2 - 2xy + 3y^2$$

$\therefore$ Area of rectangle = Product of two sides

$$= (5x^2 + 25xy + 4y^2)(2x^2 - 2xy + 3y^2)$$

$$\begin{aligned}
 &= 10x^4 - 10x^3y + 15x^2y^2 + 50x^3y - 50x^2y^2 + 75xy^3 + \\
 & \quad 8x^2y^2 - 8xy^3 + 12y^4
 \end{aligned}$$

$$= 10x^4 + 40x^3y - 27x^2y^2 + 67xy^3 + 12y^4$$