Question 1.

Using suitable identities, find the following products:

(i) $(3x + 5)(3x + 5)$

(ii) $(9y - 5)(9y-5)$

(iii) $(4x + 11y)(4x - 11y)$

(iv) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right)$

(v) $\left(\frac{2}{a} + \frac{5}{b}\right)\left(\frac{2}{a} + \frac{5}{b}\right)$

(vi) $\left(\frac{p^2}{2} + \frac{2}{q^2}\right)\left(\frac{p^2}{2} - \frac{2}{q^2}\right)$

Solution:

(i) $(3x + 5)(3x + 5)$

$= (3x + 5)^2$

$= (3x)^2 + 2 \times 3x \times 5 + (5)^2$

$= 9x^2 + 30x + 25$

$\{(a + b)^2 = a^2 + 2zb + b^2\}$

(ii) $(9y - 5)(9y - 5)$

$= (9y - 5)^2$

$= (9y)^2 - 2 \times 9y \times 5 + (5)^2$

$= 81y^2 - 90y + 25$

(iii) $(4x + 11y)(4x - 11y)$

$= (4x)^2 - (11y)^2$

$\{(a + b)(a - b) = a^2 - b^2\}$

$= 16x^2 - 121y^2$

(iii) $(4x + 11y)(4x - 11y)$

$= (4x)^2 - (11y)^2$

$\{(a + b)(a - b) = a^2 - b^2\}$

$= 16x^2 - 121y^2$

(iv) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right)$

$= \left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2$

$\{(a + b)(a - b) = a^2 - b^2\}$

$= \frac{9}{4}m^2 - \frac{4}{9}n^2$

(v) $\left(\frac{2}{a} + \frac{5}{b}\right)\left(\frac{2}{a} + \frac{5}{b}\right)$

$= \left(\frac{2}{a} + \frac{5}{b}\right)^2$

$= \left(\frac{2}{a}\right)^2 + 2 \times \frac{2}{a} \times \frac{5}{b} + \left(\frac{5}{b}\right)^2$

$\{(a + b)^2 = a^2 + 2ab + b^2\}$

$= \frac{4}{a^2} + \frac{20}{ab} + \frac{25}{b^2}$

(vi) $\left(\frac{p^2}{2}+\frac{2}{q^2}\right)\left(\frac{p^2}{2}-\frac{2}{q^2}\right)$

$$=\left(\frac{p^2}{2}\right)^2-\left(\frac{2}{q^2}\right)^2$$

$\{(a+b)(a-b)=a^2-b^2\}$

$=\frac{p^4}{4}-\frac{4}{q^4}$

Question 2.
Using the identities, evaluate the following:
(i) 81^2
(ii) 97^2
(iii) 105^2
(iv) 997^2
(v) 6.1^2
(vi) 496 × 504
(vii) 20.5 × 19.5
(viii) 9.62
Solution:
(i) $(81)^2 = (80 + 1)^2$
$= (80)^2 + 2 \times 80 \times 1 + (1)^2$ $\{(a + b)^2 = a^2 + 2ab + b^2\}$
= 6400 + 160+ 1 = 6561

(ii) $(97)^2 = (100 - 3)^2$
$= (100)^2 - 2 \times 100 \times 3 + (3)^2$ $\{(a - b)^2 = a^2 - 2ab + b^2\}$
= 10000 − 600 + 9
= 10009 − 600 = 9409

(ii) $(105)^2 = (100 + 5)^2$

$= (100)^2 + 2 \times 100 \times 5 + (5)^2 \{(a + b)^2 = a^2 + 2ab + b^2\}$

$= 10000 + 1000 + 25 = 11025$

(iv) $(997)^2 = (1000 - 3)^2$

$= (1000)^2 - 2 \times 1000 \times 3 + (3)^2 \{(a - b)^2 = a^2 - 2ab + b^2\}$

$= 1000000 - 6000 + 9$

$= 1000009 - 6000 = 994009$

(v) $(6.1)^2 = (6 + 0.1)^2$

$= (6)^2 + 2 \times 6 \times 0.1 + (0.1)^2 \{(a + b)^2 = a^2 + 2ab + b^2\}$

$= 36 + 1.2 + 0.01 = 37.21$

(vi) 496×504

$= (500 - 4)(500 + 4) \{(a + b)(a - b) = a^2 - b^2\}$

$= (500)^2 - (4)^2$

$= 250000 - 16 = 249984$

(vii) 20.5×19.5

$(20 + 0.5)(20 - 0.5) \{(a + b)(a - b) = a^2 - b^2\}$

$= (20)^2 - (0.5)^2$

$= 400 - 0.25 = 399.75$

(viii) $(9.6)^2 = (10 - 0.4)^2$

$= (10)^2 - 2 \times 10 \times 0.4 + (0.4)^2 \{(a - b)^2 = a^2 - 2ab + b^2\}$

$= 100 - 8.0 + 0.16 = 92.16$

Question 3.

Find the following squares, using the identities:

(i) $(pq + 5r)^2$ (ii) $\left(\frac{5}{2}a - \frac{3}{5}b\right)^2$

(iii) $(\sqrt{2}\,a + \sqrt{3}\,b)^2$ (iv) $\left(\frac{2x}{3y} - \frac{3y}{2x}\right)^2$

Solution:

(i) $(pq + 5r)^2$

$(pq)^2 + 2 \times pq \times 5r + (5r)^2$ $\{(a + b)^2 = a^2 + 2ab + b^2\}$

$= p^2q^2 + 10pqr + 25r^2$

(ii) $\left(\frac{5}{2}a - \frac{3}{5}b\right)^2$

$$= \left(\frac{5}{2}a\right)^2 - 2 \times \frac{5}{2}a \times \frac{3}{5}b + \left(\frac{3}{5}b\right)^2$$

$$\{(a - b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{25}{4}a^2 - 3ab \times \frac{9}{25}b^2$$

(iii) $(\sqrt{2}a + \sqrt{3}b)^2$

$= (\sqrt{2}a)^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + (\sqrt{3}b)$ $\{(a - b)^2 = a^2 - 2ab + b^2\}$

$= 2a^2 + 2\sqrt{6}ab + 3b^2$

(iv) $\left(\frac{2x}{3y}-\frac{3y}{2x}\right)^2$

$$= \left(\frac{2x}{3y}\right)^2 - 2 \times \frac{2x}{2y} \times \frac{3y}{2x} + \left(\frac{3y}{2x}\right)^2$$

$$\{(a-b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2}$$

Question 4.
Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$,
find the following products:
(i) $(x + 7)(x + 3)$
(ii) $(3x + 4)(3x - 5)$
(iii) $(p^2 + 2q)(p^2 - 3q)$
(iv) $(abc + 3)(abc - 5)$
Solution:
(i) $(x + 7)(x + 3)$
$= (x)^2 + (7 + 3)x + 7 \times 3$
$= x^2 + 10x + 21$

(ii) $(3x + 4)(3x - 5)$
$= (3x)^2 + (4 - 5)(3x) + 4 \times (-5)$
$= 9x^2 - 3x - 20$

(iii) $(P^2 + 2q)(p^2 - 3q)$
$= (p^2)^2 + (2q - 3q)p^2 + 2q \times (-3q)$
$= p^4 - p^2q - 6pq$

(iv) $(abc + 3)(abc - 5)$

$= (abc)^2 + (3 - 5)abc + 3 \times (-5)$

$= a^2b^2c^2 - 2abc - 15$

Question 5.

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, evaluate the following:

(i) 203×204

(ii) 8.2×8.7

(iii) 107×93

Solution:

(i) 203×204

$= (200 + 3)(200 + 4)$

$= (200)^2 + (3 + 4) \times 200 + 3 \times 4$

$= 40000 + 1400 + 12 = 41412$

(ii) 8.2×8.7

$= (8 + 0.2)(8 + 0.7)$

$= (8)^2 + (0.2 + 0.7) \times 8 + 0.2 \times 0.7$

$= 64 + 8 \times (0.9) + 0.14$

$= 64 + 7.2 + 0.14 = 71.34$

(iii) 107×93

$= (100 + 7)(100 - 7)$

$= (100)^2 + (7 - 7) \times 100 + 7 \times (-7)$

$= 10000 + 0 - 49 = 9951$

Question 6.

Using the identity $a^2 - b^2 = (a + b)(a - b)$, find

(i) $53^2 - 47^2$

(ii) $(2.05)^2 - (0.95)^2$

(iii) $(14.3)^2 - (5.7)^2$

Solution:

(i) $53^2 - 47^2$

$= (50 + 3)(50 - 3)$

$= (50)^2 - (3)^2$

$= 2500 - 9 = 2491$

(ii) $(2.05)^2 - (0.95)^2$

$= (2.05 + 0.95)(2.05 - 0.95)$

$= 3 \times 1.10 = 3.3$

(iii) $(14.3)^2 - (5.7)^2$

$= (14.3 + 5.7)(14.3 - 5.7)$

$= 20 \times 8.6 = 172$

Question 7.

Simplify the following:

(i) $(2x + 5y)^2 + (2x - 5y)^2$

(ii) $\left(\frac{7}{2}a - \frac{5}{2}b\right)^2 - \left(\frac{5}{2}a - \frac{7}{2}b\right)^2$

(iii) $(p^2 - q^2r)^2 + 2p^2q^2r$

Solution:

(i) $(2x + 5y)^2 + (2x - 5y)^2$

$= (2x)^2 + 2 \times 2x \times 5y + (5y)^2 + (2x)^2 - 2 \times 2x \times 5y + (5y)^2$

$= 4x^2 + 20xy + 25y^2 + 4x^2 - 20xy + 25y^2$

$= 8x^2 + 50y^2$

(ii) $\left(\frac{7}{2}a - \frac{5}{2}b\right)^2 - \left(\frac{5}{2}a - \frac{7}{2}b\right)^2$

$$= \left[\left(\frac{7}{2}a\right)^2 - 2 \times \frac{7}{2}a \times \frac{5}{2}b - \left(\frac{5}{2}b\right)^2\right]$$

$$- \left[\left(\frac{5}{2}a\right)^2 - 2 \times \frac{5}{2}a \times \frac{7}{2}b + \left(\frac{7}{2}b\right)^2\right]$$

$$= \left[\frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2\right]$$

$$- \left[\frac{25}{4}a^2 - \frac{35}{2}ab + \frac{49}{4}b^2\right]$$

$$= \frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2 - \frac{25}{4}a^2$$

$$+ \frac{35}{2}ab - \frac{49}{4}b^2$$

$$= \frac{49}{4}a^2 - \frac{25}{4}a^2 + \frac{25}{4}b^2 - \frac{49}{4}a^2$$

$$= \frac{24}{4}a^2 + \frac{-24}{4}b^2 = 6a^2 - 6b^2$$

(iii) $(p^2 - q^2r)^2 + 2p^2q^2r$ $\{(a - b)^2 = a^2 - 2ab + b^2\}$

$= (p^2)^2 - 2 \times p^2 \times q^2r + (q^2r)^2 + 2p^2q^2r$

$= p^4 - 2p^2q + q^4r^2 + 2p^2q^2r$

$= p^4 + q^4r^2$

Question 8.

Show that:

(i) $(4x + 7y)^2 - (4x - 7y)^2 = 112xy$

(ii) $\left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq = \frac{9}{49}p^2 + \frac{49}{36}q^2$

(iii) $(p - q)(p + q) + (q - r)(q + r) + (r - p)(r + p) = 0$

Solution:

(i) $(4x + 7y)^2 - (4x - 7y)^2 = 112xy$

LHS $= (4x + 7y)^2 - (4x - 7y)^2$

$= [(4x)^2 + 2 \times 4x \times 7y + (7y)^2]$

$- [(4x)^2 - 2 \times 4x + 7y + (7y)^2]$

$\{\because (a \pm b)^2 = a^2 \pm 2ab + b^2\}$

$= (16x^2 + 56xy + 49y^2) - (16x^2 - 56xy + 49y^2)$

$=- 16x^2 + 56xy + 49y^2 - 16x^2 + 56xy - 49y^2$

$= 112xy =$ RHS

$$(ii)\ \left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq = \frac{9}{49}p^2 + \frac{49}{36}q^2$$

$$\text{LHS} = \left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq$$

$$= \left(\frac{3}{7}p\right)^2 - 2 \times \frac{3}{7}p \times \frac{7}{6}q + \left(\frac{7}{6}q\right)^2 + pq$$

$$\{(a-b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{9}{49}p^2 - pq + \frac{49}{36}q^2 + pq$$

$$= \frac{9}{49}p^2 + \frac{49}{36}q^2 = \text{RHS}$$

(iii) $(p - q)(p + q) + (q - r)(q + r) + (r - p)(r + p) = 0$

LHS = $(p - q)(p + q) + (q - r)(q + r) + (r - p)(r + p)$

$= p^2 - q^2 + q^2 - r^2 + r^2 - p^2$

$\{(a + b)(a - b) = a^2 - b^2\}$

= 0 = RHS

Question 9.

If $x + \frac{1}{x} = 2$, evaluate:

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Solution:

$x + \frac{1}{x} = 2$

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Squaring both sides,

$$\left(x + \frac{1}{x}\right)^2 = (2)^2$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 4$$

$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 4 \Rightarrow x^2 + \frac{1}{x^2} = 4 - 2 = 2$$

$$\therefore x^2 + \frac{1}{x^2} = 2$$

Again squaring, we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (2)^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 4$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 4$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 4 - 2 = 2$$

Hence, $x^4 + \frac{1}{x^4} = 2$

Question 10.

If x = $\frac{1}{x}$ = 7, ecaluate:

(*i*) $x^2 + \frac{1}{x^2}$ (*ii*) $x^4 + \frac{1}{x^4}$

Solution:

x = $\frac{1}{x}$ = 7

(*i*) $x^2 + \frac{1}{x^2}$ (*ii*) $x^4 + \frac{1}{x^4}$

Squaring both sides,

$$\left(x - \frac{1}{x}\right)^2 = (7)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x + \frac{1}{x} = 49$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 49$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 49 + 2 = 51$$

$$\therefore x^2 + \frac{1}{x^2} = 51$$

Again squaring,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (51)^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 2601$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 2601$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 2601 - 2 = 2599$$

Question 11.
If $x^2 + \frac{1}{x^2} = 23$, evaluate:

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$

Solution:
$x^2 + \frac{1}{x^2} = 23$

(i) $x + \frac{1}{x}$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 23 + 2 = 25$$

$$\left(x + \frac{1}{x}\right) = (\pm 5)^2$$

$$\therefore x + \frac{1}{x} = \pm 5$$

(ii) $x - \frac{1}{x}$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 23 - 2 = 21$$

$$\left(x - \frac{1}{x}\right) = (\pm\sqrt{21})^2$$

$$\therefore x - \frac{1}{x} = \pm\sqrt{21}$$

Question 12.

If a + b = 9 and = 10, find the value of $a^2 + b^2$.

Solution:

a + b = 9, ab = 10

a + b = 9

Squaring both sides,

$(a + b)^2 = (9)$

$\Rightarrow a^2 + b^2 + 2ab = 81$

$\Rightarrow a^2 + b^2 + 2 \times 10 = 81$

$\Rightarrow a^2 + b^2 + 20 = 81$

$\Rightarrow a^2 + b^2 = 81 - 20 = 61$

$\therefore a^2 + b^2 = 61$

Question 13.

If $a - b = 6$ and $a^2 + b^2 = 42$, find the value of

Solution:

$a - b = 6, a^2 + b^2 = 42$

$a - b = 6$

Squaring both sides,

$(a - b)^2 = (6)^2$

$\Rightarrow a^2 + b^2 - 2ab = 36$

$\Rightarrow 42 - 2ab = 36$

$\Rightarrow 2ab = 42 - 36 = 6$

$\Rightarrow ab = \frac{6}{2} = 3$

$\therefore ab = 3$

Question 14.

If $a^2 + b^2 = 41$ and $ab = 4$, find the values of

(i) $a + b$

(ii) $a - b$

Solution:

$a^2 + b^2 = 41, ab = 4$

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

$= 41 + 2 \times 4 = 41 + 8 = 49$

$= (\pm 7)^2$

$\therefore a + b = \pm 7$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

$= 41 - 2 \times 4 = 41 - 8 = 33$

$\therefore a - b = \pm\sqrt{33}$