

Factorise the following (1 to 11) polynomials:

Question 1.

(i) $x^2 + xy - x - y$

(ii) $y^2 - yz - 5y + 5z$

Solution:

(i) $x^2 + xy - x - y$

$$= x(x + y) - 1(x + y) = (x + y)(x - 1)$$

(ii) $y^2 - yz - 5y + 5z$

$$= y(y - z) - 5(y - z)$$

$$= (y - z)(y - 5)$$

Question 2.

(i) $5xy + 7y - 5y^2 - 7x$

(ii) $5p^2 - 8pq - 10p + 16q$

Solution:

(i) $5xy + 7y - 5y^2 - 7x$

$$= 5xy - 5y^2 + 7y - 7x$$

$$= 5y(x - y) - 7(x - y)$$

$$= (x - y)(5y - 1)$$

(ii) $5p^2 - 8pq - 10p + 16q$

$$= 5p^2 - 10p - 8pq + 16q$$

$$= 5p(p - 2) - 8q(p - 2)$$

$$= (p - 2)(5p - 8q)$$

$$= (5p - 8q)(p - 2)$$

Question 3.

(i) $a^2b - ab^2 + 3a - 3b$

(ii) $x^3 - 3x^2 + x - 3$

Solution:

(i) $a^2b - ab^2 + 3a - 3b$

$$= ab(a - b) + 3(a - b) = (a - b)(ab + 3)$$

(ii) $x^3 - 3x^2 + x - 3$

$$= x^2(x - 3) + 1(x - 3)$$

$$= (x - 3)(x^2 + 1)$$

Question 4.

(i) $6xy^2 - 3xy - 10y + 5$

(ii) $3ax - 6ay - 8by + 4bx$

Solution:

(i) $6xy^2 - 3xy - 10y + 5$

$$3xy(2y - 1) - 5(2y - 1)$$

$$= (2y - 1)(3xy - 5)$$

(ii) $3ax - 6ay - 8by + 4bx$

$$= 3ax - 6ay + 4bx - 8by$$

$$= 3a(x - 2y) + 4b(x - 2y)$$

$$= (x - 2y)(3a + 4b)$$

Question 5.

(i) $x^2 + xy(1 + y) + y^3$

(ii) $y^2 - xy(1 - x) - x^3$

Solution:

(i) $x^2 + xy(1 + y) + y^3$

$$= x^2 + xy + xy^2 + y^3$$

$$= x(x + y) + y^2(x + y)$$

$$= (x + y)(x + y^2)$$

$$\begin{aligned}
 & \text{(ii) } y^2 - xy(1 - x) - x^3 \\
 &= y^2 - xy + x^2y - x^3 \\
 &= y(y - x) + x^2(y - x) \\
 &= (y - x)(y + x^2)
 \end{aligned}$$

Question 6.

$$\begin{aligned}
 & \text{(i) } ab^2 + (a - 1)b - 1 \\
 & \text{(ii) } 2a - 4b - xa + 2bx
 \end{aligned}$$

Solution:

$$\begin{aligned}
 & \text{(i) } ab^2 + (a - 1)b - 1 \\
 &= ab^2 + ab - b - 1 \\
 &= ab(b + 1) - 1(b + 1) \\
 &= (b + 1)(ab - 1)
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii) } 2a - 4b - xa + 2bx \\
 &= 2(a - 2b) - x(a - 2b) \\
 &= (a - 2b)(2 - x)
 \end{aligned}$$

Question 7.

$$\begin{aligned}
 & \text{(i) } 5ph - 10qk + 2rph - 4qrk \\
 & \text{(ii) } x^2 - x(a + 2b) + 2a^2
 \end{aligned}$$

Solution:

$$\begin{aligned}
 & \text{(i) } 5ph - 10qk + 2rph - 4qrk \\
 &= 5(ph - 2qk) + 2r(ph - 2qk) \\
 &= (ph - 2qk)(5 + 2r)
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii) } x^2 - x(a + 2b) + 2ab \\
 &= x^2 - xa - 2bx + 2ab \\
 &= x(x - a) - 2b(x - a) \\
 &= (x - a)(x - 2b)
 \end{aligned}$$

Question 8.

(i) $ab(x^2 + y^2) - xy(a^2 + b^2)$

(ii) $(ax + by)^2 + (bx - ay)^2$

Solution:

(i) $ab(x^2 + y^2) - xy(a^2 + b^2)$

$$= abx^2 + aby^2 - a^2xy - b^2xy$$

$$= (abx^2 - b^2xy) + (aby^2 - a^2xy)$$

$$= bx(ax - by) - ay(ax - by)$$

$$= (ax - by)(bx - ay)$$

(ii) $(ax + by)^2 + (bx - ay)^2$

$$= (a^2x^2 + b^2y^2 + 2abxy) + (b^2x^2 + a^2y^2 - 2abxy)$$

$$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$$

$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$

$$= a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$$

$$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$$

$$= (a^2 + b^2)(x^2 + y^2)$$

Question 9.

(i) $a^3 + ab(1 - 2a) - 2b^2$

(ii) $3x^2y - 3xy + 12x - 12$

Solution:

(i) $a^3 + ab - 2a^2b - 2b^2$

$$= a^3 + ab - 2a^2b - 2b^2$$

$$= a(a^2 + b) - 2b(a^2 + b)$$

$$= (a^2 + b)(a - 2b)$$

(ii) $3x^2y - 3xy + 12x - 12$

$$= 3(x^2y - xy + 4x - 4)$$

$$= 3[xy(x - 1) + 4(x - 1)]$$

$$= 3(x - 1)(xy + 4)$$

Question 10.

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$

(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

Solution:

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$

$$= ab(a + b) - bc(a + b) + xy(a + b)$$

$$= (a + b)(ab - bc + xy)$$

(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

$$= x^2(a - b) + y^2(a - b) + z^2(a - b)$$

$$= (a - b)(x^2 + y^2 + z^2)$$

Question 11.

(i) $x - 1 - (x - 1)^2 + ax - a$

(ii) $ax + a^2x + aby + by - (ax + by)^2$

Solution:

(i) $x - 1 - (x - 1)^2 + ax - a$

$$= (x - 1) - (x - 1)^2 + a(x - 1)$$

$$= (x - 1)[1 - (x - 1) + a]$$

$$= (x - 1)(1 - x + 1 + a)$$

$$= (x - 1)(2 - x + a)$$

(ii) $ax + a^2x + aby + by - (ax + by)^2$

$$= (ax + by) + (a^2x + aby) - (ax + by)^2$$

$$= (ax + by) + a(ax + by) - (ax + by)^2$$

$$= (ax + by)[1 + a - (ax + by)]$$

$$= (ax + by)(1 + a - ax - by)$$