

Question 1.

Factorise the following expressions using algebraic identities:

$$(i) x^2 - 12x + 36$$

$$(ii) 36p^2 - 60pq + 25q^2$$

$$(iii) 9y^2 + 66xy + 121y^2$$

$$(iv) a^4 + 6a^2b^2 + 9b^4$$

$$(v) x^2 + \frac{1}{x^2} + 2$$

$$(vi) x^2 + x + \frac{1}{4}$$

Solution:

Using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(i) y^2 - 12x + 36$$

$$= (x)^2 - 2 \times x \times 6 + (6)^2$$

$$= (x - 6)^2$$

$$(ii) 36p^2 - 60pq + 25q^2$$

$$= (6p)^2 - 2 \times 6p \times 5q + (5q)^2$$

$$= (6p - 5q)^2$$

$$(iii) 9x^2 + 66xy + 121y^2$$

$$= (3x)^2 + 2 \times 3x \times 11y + (11y)^2$$

$$= (3x + 11y)^2$$

$$(iv) a^4 + 6a^2b^2 + 9b^4$$

$$= (a^2)^2 + 2 \times a^2 \times 3b^2 + (3b^2)^2$$

$$= (a^2 + 3b^2)^2$$

$$(v) x^2 + \frac{1}{x^2} + 2$$

$$= (x)^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$$

$$= \left(x + \frac{1}{x}\right)^2$$

$$(vi) x^2 + x + \frac{1}{4}$$

$$= (x)^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= \left(x + \frac{1}{2}\right)^2$$

Factorise the following (2 to 13) expressions:

Question 2.

$$(i) 4p^2 - 9$$

$$(ii) 4x^2 - 169y^2$$

Solution:

$$(i) 4p^2 - 9$$

$$= (2p)^2 - (3)^2$$

$$= (2p + 3)(2p - 3)$$

$$(ii) 4x^2 - 169y^2$$

$$= (2x)^2 - (13y)^2$$

$$= (2x + 13y)(2x - 13y)$$

Question 3.

$$(i) 9x^2y^2 - 25$$

$$(ii) 16x^2 - \frac{1}{144}$$

Solution:

$$\begin{aligned}(\text{i}) \quad & 9x^2y^2 - 25 \\&= (3xy)^2 - (5)^2 \\&= (3xy + 5)(3xy - 5)\end{aligned}$$

$$(\text{ii}) \quad 16x^2 - \frac{1}{144} = (4x)^2 - \left(\frac{1}{12}\right)^2$$

$$= \left(4x + \frac{1}{12}\right) \left(4x - \frac{1}{12}\right)$$

Question 4.

$$\begin{aligned}(\text{i}) \quad & 20x^2 - 45y^2 \\(\text{ii}) \quad & \frac{9}{16} - 25a^2b^2\end{aligned}$$

Solution:

$$\begin{aligned}(\text{i}) \quad & 20x^2 - 45y^2 \\&= 5(4x^2 - 9y^2) \\&= 5[(2x)^2 - (3y)^2] \\&= 5(2x + 3y)(2x - 3y)\end{aligned}$$

$$(\text{ii}) \quad \frac{9}{16} - 25a^2b^2$$

$$\begin{aligned}&= \left(\frac{3}{4}\right)^2 - (5ab)^2 \\&= \left(\frac{3}{4} + 5ab\right) \left(\frac{3}{4} - 5ab\right)\end{aligned}$$

Question 5.

$$\begin{aligned}(\text{i}) \quad & (2a + 3b)^2 - 16c^2 \\(\text{ii}) \quad & 1 - (b - c)^2\end{aligned}$$

Solution:

$$\begin{aligned}(i) \quad & (2a + 3b)^2 - 16c^2 \\&= (2a + 3b)^2 - (4c)^2 \\&= (2a + 3b + 4c)(2a + 3b - 4c)\end{aligned}$$

$$\begin{aligned}(ii) \quad & 1 - (b - c)^2 \\&= (1)^2 - (b - c)^2 \\&= [1 + b - c][1 - (b - c)] \\&= (1 + b - c)(1 - b + c)\end{aligned}$$

Question 6.

$$\begin{aligned}(i) \quad & 9(x + y)^2 - x^2 \\(ii) \quad & (2m + 3n)^2 - (3m + 2n)^2\end{aligned}$$

Solution:

$$\begin{aligned}(i) \quad & 9(x + y)^2 - x^2 \\&= [3(x + y)]^2 - [x]^2 \\&= [3(x + y) + x][3(x + y) - x] \\&= (3x + 3y + x)(3x + 3y - x) \\&= (4x + 3y)(2x + 3x)\end{aligned}$$

$$\begin{aligned}(ii) \quad & (2m + 3n)^2 - (3m + 2n)^2 \\&= (4m^2 + 9n^2 + 12mn) - (9m^2 + 4n^2 + 12mn) \\&= 4m^2 + 9n^2 + 12mn - 9m^2 - 4m^2 - 12mn \\&= 4m^2 + 9n^2 - 9m^2 - 4n^2 \\&= -5m^2 + 5n^2 = 5(n^2 - m^2) \\&= 5(m + n)(n - m)\end{aligned}$$

Question 7.

$$\begin{aligned}(i) \quad & 25(a + b)^2 - 16(a - b)^2 \\(ii) \quad & 9(3x + 2)^2 - 4(2x - 1)^2\end{aligned}$$

Solution:

$$\begin{aligned}(i) & 25(a+b)^2 - 16(a-b)^2 \\&= [5(a+b)]^2 - [4(a-b)]^2 \\&= (5a+5b)^2 - (4a-4b)^2 \\&= [(5a+5b)^2 + (4a-4b)][(5a+5b) - (4a-4b)] \\&= (5a+5b+4a-4b)(5a+5b-4a+4b) \\&= (9a+ft)(a+9ft)\end{aligned}$$

$$\begin{aligned}(ii) & 9(3x+2)^2 - 4(2x-1)^2 \\&= [3(3x+2)]^2 - [2(2x-1)]^2 \\&= (9x+6)^2 - (4x-2)^2 \\&= [(9x+6) + (4x-2)][(9x+6) - (4x-2)] \\&= (9x+6+4x-2)(9x+6-4x+2) \\&= (13x+4)(5x+8)\end{aligned}$$

Question 8.

$$\begin{aligned}(i) & x^3 - 25x \\(ii) & 63p^2q^2 - 7\end{aligned}$$

Solution:

$$\begin{aligned}(i) & x^3 - 25x \\&= x(x^2 - 25) = x[(x)^2 - (5)^2] \\&= x(x+5)(x-5)\end{aligned}$$

$$\begin{aligned}(ii) & 63p^2q^2 - 7 \\&= 7(9p^2q^2 - 1) \\&= 7[(3pq)^2 - (1)^2] \\&= 7(3pq+1)(3pq-1)\end{aligned}$$

Question 9.

$$\begin{aligned}(i) & 32a^2b - 72b^3 \\(ii) & 9(a+b)^3 - 25(a+b)\end{aligned}$$

Solution:

$$\begin{aligned} \text{(i)} & 32a^2b - 72b^3 \\ & = 8b(4a^2 - 9b^2) \Rightarrow 8b[(2a)^2 - (3b)^2] \\ & = 8b(2a + 3b)(2a - 3b) \end{aligned}$$

$$\begin{aligned} \text{(ii)} & 9(a+b)^3 - 25(a+b) \\ & = (a+b)[9(a+b)^2 - 25] \\ & = (a+b)[\{3(a+b)\}^2 - (5)^2] \\ & = (a+6)[(3a+3b)^2 - (5)^2] \\ & = (a+b)[(3a+3b+5)(3a+3b-5)] \\ & = (a+b)(3a+3b+5)(3a+3b-5) \end{aligned}$$

Question 10.

$$\begin{aligned} \text{(i)} & x^2 - y^2 - 2y - 1 \\ \text{(ii)} & p^2 - 4pq + 4q^2 - r^2 \end{aligned}$$

Solution:

$$\begin{aligned} \text{(i)} & x^2 - y^2 - 2y - 1 \\ & = x^2 - (y^2 + 2y + 1) \\ & = (x)^2 - (y + 1)^2 \\ & = [x + (y + 1)][x - (y + 1)] \\ & = (x + y + 1)(x - y - 1) \end{aligned}$$

$$\begin{aligned} \text{(ii)} & p^2 - 4pq + 4q^2 - r^2 \\ & = (p)^2 - 2 \times p \times 2q + (2q)^2 - r^2 \\ & \{\because (a - b)^2 = a^2 - 2ab + b^2\} \\ & a^2 - b^2 = (a + b)(a - b) \\ & = (p - 2q)^2 - (r)^2 \\ & = (p - 2q + r)(p - 2q - r) \end{aligned}$$

Question 11.

$$(i) 9x^2 - y^2 + 4y - 4$$

$$(ii) 4a^2 - 4b^2 + 4a + 1$$

Solution:

$$(i) 9x^2 - y^2 + 4y - 4$$

$$= 9x^2 - (y^2 - 4y + 4)$$

$$= 9x^2 - (y - 2)^2$$

$$= (3x)^2 (y - 2)^2$$

$$= \{3x + (y - 2)\} \{3x - (y - 2)\}$$

$$= (3x + y - 2) (3x - y + 2)$$

$$(ii) 4a^2 - 4b^2 + 4a + 1$$

$$= (4a^2 + 4a + 1) - 4b^2$$

$$= (2a + 1)^2 - (2b)^2$$

$$= (2a + 2b + 1) (2a - 2b + 1)$$

Question 12.

$$(i) 625 - p^4$$

$$(ii) 5y^5 - 405y$$

Solution:

$$(i) 625 - p^4$$

$$= (25)^2 - (p^2)^2$$

$$= (25 + p^2) (25 - p^2)$$

$$= (25 + p^2) [(5)^2 - (p)^2]$$

$$= (25 + p^2) (5 + p) (5 - p)$$

$$\begin{aligned}
 \text{(ii)} \quad & 5y^5 - 405y \\
 = & 5y(y^4 - 81) \\
 = & 5y [(y^2)^2 - (9)^2] \\
 = & 5y (y^2 + 9) (y^2 - 9) \\
 = & 5y (y^2 + 9) [(y)^2 - (3)^2] \\
 = & 5y (y^2 + 9) (y + 3) (y - 3)
 \end{aligned}$$

Question 13.

$$\begin{aligned}
 \text{(i)} \quad & x^4 - y^4 + x^2 - y^2 \\
 \text{(ii)} \quad & 64a^2 - 9b^2 + 42bc - 49c^2
 \end{aligned}$$

Solution:

$$\begin{aligned}
 \text{(i)} \quad & x^4 - y^4 + x^2 - y^2 \\
 = & [(x^2)^2 - (y^2)^2] + (x^2 - y^2) \\
 \{a^2 - b^2 = (a + b)(a - b)\} \\
 = & (x^2 + y^2)(x^2 - y^2) + 1(x^2 - y^2) \\
 = & (x^2 - y^2)(x^2 + y^2 + 1) \\
 = & (x + y)(x - y)(x^2 + y^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & 64a^2 - 9b^2 + 42bc - 49c^2 \\
 = & 64a^2 - [9b^2 - 42bc + 49c^2] \\
 = & (8a)^2 - [(3b)^2 - 2 \times 3b \times 7c + (7c)^2] \\
 \{\because a^2 + b^2 - 2ab = (a - b)^2\} \\
 a^2 - b^2 = (a + b)(a - b) \\
 = & (8a)^2 - (3b - 7c)^2 \\
 = & (8a + 3b - 7c)(8a - 3b + 7c)
 \end{aligned}$$