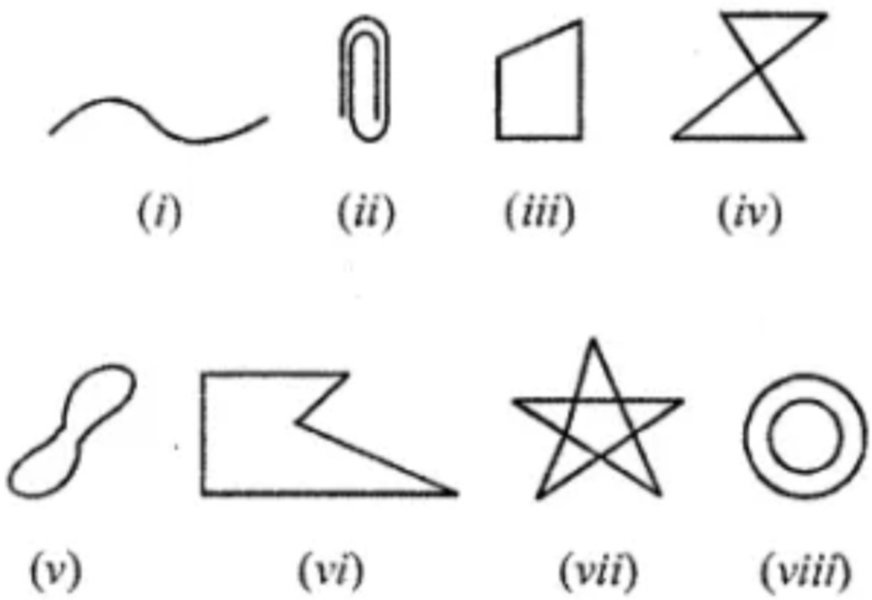
Question 1.

Some figures are given below.



Classify each of them on the basis of the following:

(a) Simple curve

(b) Simple closed curve

(c) Polygon

(d) Convex polygon

(e) Concave polygon

Solution:

(a) (i), (ii), (iii), (v) and (vi) are simple curves.

(b) (iii), (v), (vi) are simple closed curves.

(c) (iii) and (vi) are polygons.

(d) (iii) is a convex polygon.

(e) (v) is a concave polygon.

Question 2.

How many diagonals does each of the following have?

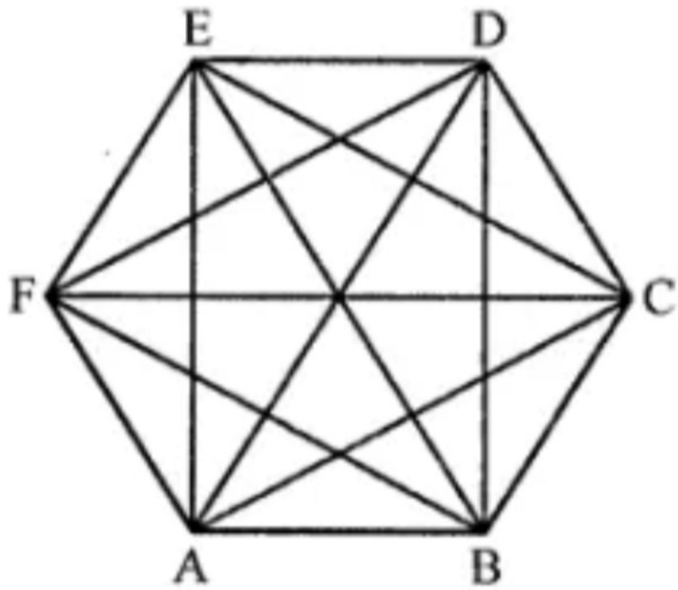
(a) A convex quadrilateral

(b) A regular hexagon

Solution:

(a) A convex quadrilateral: It has two diagonals.

(b) A regular hexagon: It has 9 diagonals as shown.



Question 3.

Find the sum of measures of all interior angles of a polygon with number of sides:

(i) 8

(ii) 12

Solution:

(i) Sum of measures of all interior angles of 8–sided polygon = $(2n - 4) \times 90°$

$= (2 \times 8 - 4) \times 90°$

$= 12 \times 90° = 1080°$

(ii) Sum of measures of all interior angles of 12–sided polygon = $(2n - 4) \times 90°$

$= (2 \times 12 - 4) \times 90°$

$= 18 \times 90° = 1800°$

Question 4.

Find the number of sides of a regular polygon whose each exterior angles has a measure of

(i) 24°

(ii) 60°

(iii) 72°

Solution:

(i) Let number of sides of the polygon = n

Each exterior angle = 24°

$\therefore$ n = $\frac{360°}{24°}$ = 15 sides

$\therefore$ Polygon is of 15 sides.

(ii) Each interior angle of the polygon = 60°

Let number of sides of the polygon = n

$\therefore$ n = $\frac{360°}{60°}$ = 6

$\therefore$ Number of sides = 6

(iii) Each interior angle of the polygon = 72°

Let number of sides of the polygon = n

$\therefore$ n = $\frac{360°}{72°}$ = 5

$\therefore$ Number of sides = 5

Question 5.

Find the number of sides of a regular polygon if each of its interior angles is

(i) 90°

(ii) 108°

(iii) 165°

Solution:

(i) Each interior angle = 90°

Let number of sides of the regular polgyon = n

$\therefore$ 90° = $\frac{2n-4}{n}$ × 90°

$\Rightarrow \frac{2n-4}{n} = \frac{90°}{90°}$ = 1

⇒ 2n − 4 = n

⇒ 2n − n = 4

⇒ n = 4

⇒ n = 4

∴ It is a square.

(ii) Each interior angle = 108°

Let number of sides of the regular polygon = n

∴ 108° = $\frac{2n-4}{n}$ × 90°

⇒ $\frac{2n-4}{n} = \frac{108°}{90°} = \frac{6}{5}$

⇒ 10n − 20 = 6n ⇒ 10n − 6n = 20

⇒ 4n = 20

⇒ n = $\frac{20}{4}$ = 5

∴ It is a pentagon.

(iii) Each interior angle = 165°

Let number of sides of the regular polygon = n

∴ 165° = $\frac{2n-4}{n}$ × 90°

⇒ $\frac{2n-4}{n} = \frac{165°}{90°} = \frac{11}{6}$

⇒ 12n − 24 = 11n

⇒ 12n − 11n = 24

⇒ n = 24

∴ It is 24–sided polygon.

Question 6.

Find the number of sides in a polygon if the sum of its interior angles is:

(i) 1260°

(ii) 1980°

(iii) 3420°

Solution:

We know that, sum of interior angles of polygon is given by (2n − 4) at right angles.

(i) 1260°

$\therefore$ 1260 = (2n – 4) × 90

$\Rightarrow \frac{1260}{90}$ = 2n – 4

$\Rightarrow$ 14 = 2n – 4

$\Rightarrow$ n = 9

(ii) 1980°

$\therefore$ 1980 = (2n – 4) × 90

$\Rightarrow \frac{1980}{90}$ = 2n – 4

$\Rightarrow$ 22 = 2n – 4

$\Rightarrow$ n = 13.

(iii) 3420°

$\therefore$ 3420 = (272 – 4) × 90 3420

$\Rightarrow \frac{3420}{90}$ = 2n – 4

$\Rightarrow$ 38 = 2n – 4

$\Rightarrow$ n = 21

Question 7.

If the angles of a pentagon are in the ratio 7 : 8 : 11 : 13 : 15, find the angles.

Solution:

Ratio in the angles of a polygon = 7 : 8 : 11 : 13 : 15

Sum of angles of a pentagon = (2n – 4) × 90°

= (2 × 5 – 4) × 90°

= 6 × 90° = 540°

Let the angles of the pentagon be

7x, 8x, 11x, 13x, 15x

$\therefore$ 7x + 8x + 11x + 13x + 15x = 540°

$\Rightarrow$ 54x = 540° $\Rightarrow$ x = $\frac{540°}{54}$ = 10°

$\therefore$ Angles are 7 × 10° = 70°, 8 × 10° = 80°,

11 × 10° = 110°, 13 × 10° = 130° and 15 × 10°= 150°

$\therefore$ Angles are 70°, 80°, 110°, 130° and 150°

Question 8.

The angles of a pentagon are x°, (x – 10)°, (x + 20)°, (2x – 44)° and (2x – 70°) Calculate x.

Solution:

Angles of a pentaon are x°, (x – 10)°, (x + 20)°, (2x – 44)° and (2x – 70°)

But sum of angles of a pentagon

$= (2n - 4) \times 90°$

$= (2 \times 5 - 4) \times 90°$

$= 6 \times 90° = 540°$

$\therefore x + x - 10° + x + 20° + 2x - 44° + 2x - 70° = 540°$

$\Rightarrow 7x - 104° = 540°$

$\Rightarrow 7x = 540° + 104° = 644°$

$\Rightarrow x = \frac{644°}{7} = 92°$

$\therefore x = 92°$

Question 9.

The exterior angles of a pentagon are in ratio 1 : 2 : 3 : 4 : 5. Find all the interior angles of the pentagon.

Solution:

Let the exterior angles of the pentagon are x, 2x, 3x, 4x and 5x.

We know that sum of exterior angles of polygon is 360°.

$\therefore x + 2x + 3x + 4x + 5x = 360°$

$\Rightarrow 15x = 360°$

$\Rightarrow x = \frac{360°}{15}$

$\Rightarrow x = 24°$

$\therefore$ Exterior angles are 24°, 48°, 72°, 96°, 120°

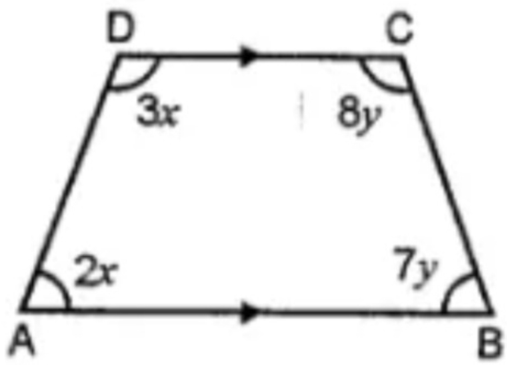
Interior angles are 180° – 24°, 180° – 48°, 180° – 72°, 180° – 96°, 180° − 120° i.e. 156°, 132°, 108°, 84°, 60°.

Question 10.

In a quadrilateral ABCD, AB || DC. If ∠A : ∠D = 2:3 and ∠B : ∠C = ∠7 : 8, find the measure of each angle.

Solution:

As AB || CD



∠A + ∠D = 180° and ∠B + ∠C = 180°

⇒ 2x + 3x = 180° and 7y + 8y = 180°

5x = 180° and 15y = 180°

x = 36° and y = 12°

∴ ∠A = 2 × 36 = 72°

and ∠D = 3 × 36 = 108°

∠B = 7y = 7 × 12 = 84°

and ∠C = 8y = 8 × 12 = 96°

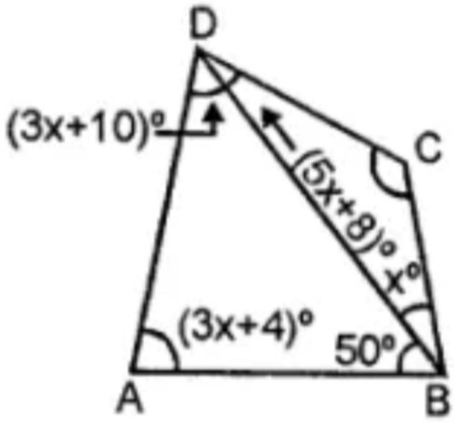
Question 11.

From the adjoining figure, find

(i) x

(ii) ∠DAB

(iii) ∠ADB



Solution:

(i) ABCD is a quadrilateral

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow (3x + 4) + (50 + x) + (5x + 8) + (3x + 10) = 360$

$\Rightarrow 3x + 4 + 50 + x + 5x + 8 + 3x + 10 = 360°$

$\Rightarrow 12x + 72 = 360°$

$\Rightarrow 12x = 288$

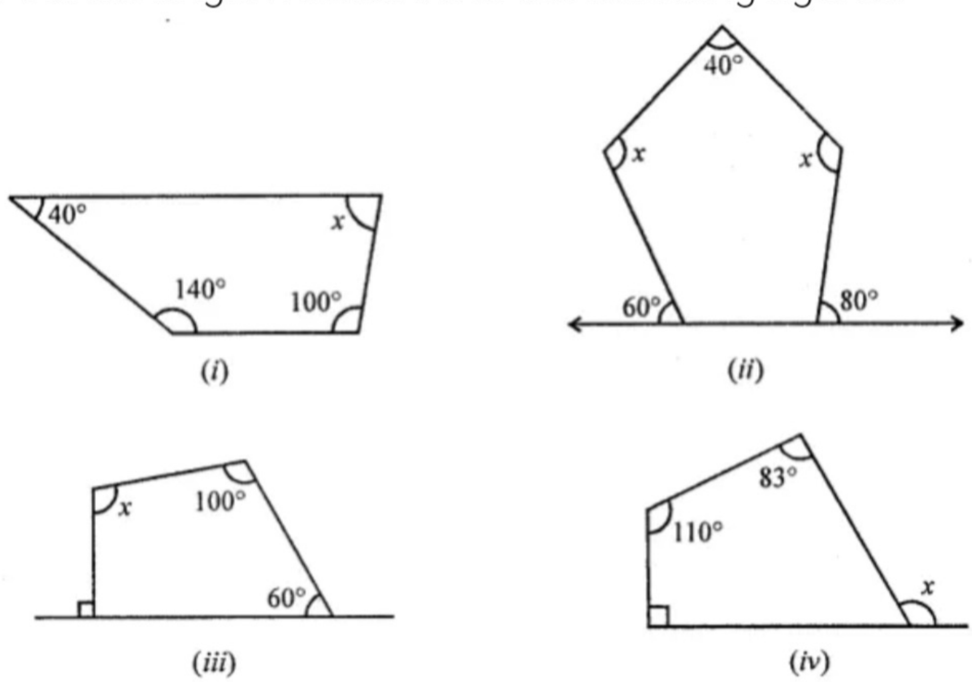
$\Rightarrow x = 24$

(ii) $\angle DAB = (3x + 4) = 3 \times 24 + 4 = 76°$

(iii) $\angle ADB = 180° - (76° + 50°) = 54°$ ($\because$ ABD is a Δ)

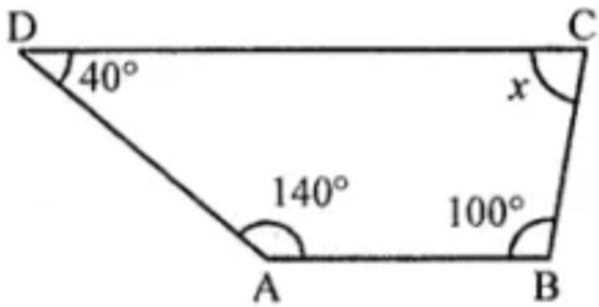
Question 12.

Find the angle measure x in the following figures:



Solution:

(i) In quadrilateral three angles are 40°, 140° and 100°



But sum of Four angles = 360°

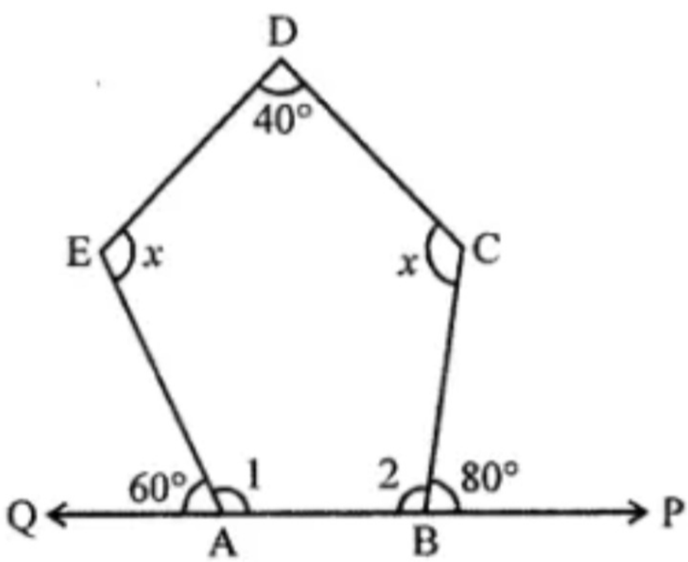
$\Rightarrow 40° + 140° + 100° + x = 360°$

$\Rightarrow 280° + x = 360°$

$\Rightarrow x = 360° - 280° = 80°$

(ii) In the given figure, ABCDE is a pentagon.

Where side AB is produced to both sides0



$\angle 1 + 60° = 180°$ (Linear pair)

$\angle 1 = 180° - 60° = 120°$

Similarly $\angle 2 + 80° = 180°$

$\therefore \angle 2 = 180° - 80° = 100°$

Now, sum of angles of a pentagon = $(2n - 4) \times 90°$

$= (2 \times 5 - 4) \times 90° = 6 \times 90° = 540°$

$\therefore \angle A + \angle B + \angle C + \angle D + \angle E = 540°$

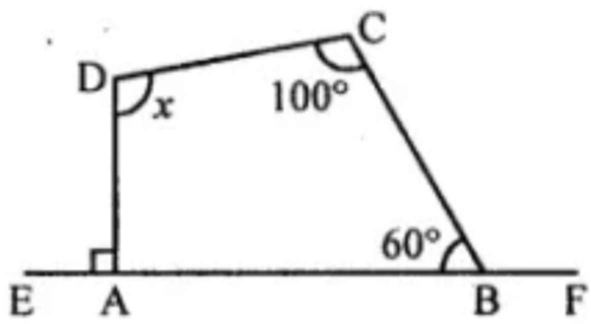
$\Rightarrow 120° + 100° + x + 40° + x = 540°$

$\Rightarrow 260° + 2x = 540°$

$\Rightarrow 2x = 540° - 260° = 280°$

$\Rightarrow x = \frac{280°}{2} = 140°$

(iii) In the given figure, ABCD is a quadrilateral whose side AB is produced is both sides ∠A = 90°



But ∠A + ∠B + ∠C + ∠D = 360°

(Sum of angles of a quadrilateral)

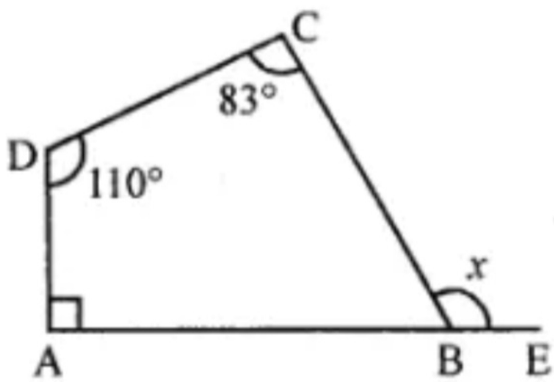
$\Rightarrow 90° + 60° + 110° + x = 360°$

$\Rightarrow 260° + x = 360°$

$\Rightarrow x = 360° - 260° = 100°$

$\therefore x = 100°$

(iv) In the given figure, ABCD is a quadrilateral whose side AB is produced to E.



∠A = 90°, ∠C = 83°, ∠D = 110°

∠B + x = 180° (Lienar pair)

∠B = 180° − x

But ∠A + ∠B + ∠C + ∠D = 360°

$\Rightarrow 90° + (180° - x) + 83° + 110° = 360°$

(Sum of angles of a quadrilateral)

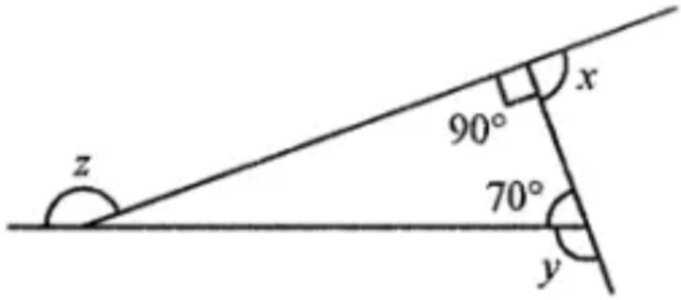
$\Rightarrow 283° + 180° - x = 360°$

$\Rightarrow x = 283° + 180° - 360°$

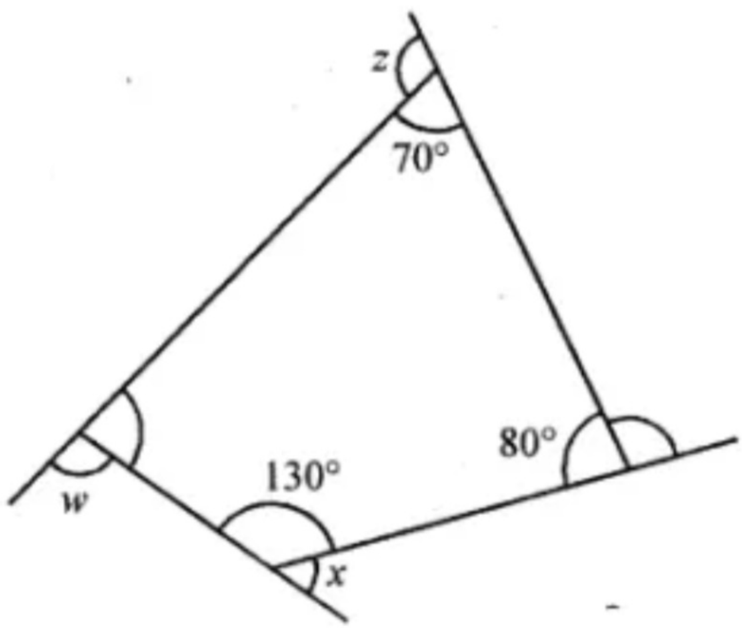
$\Rightarrow x = 463° - 360° = 103°$

Question 13.

(i) In the given figure, find x + y + z.

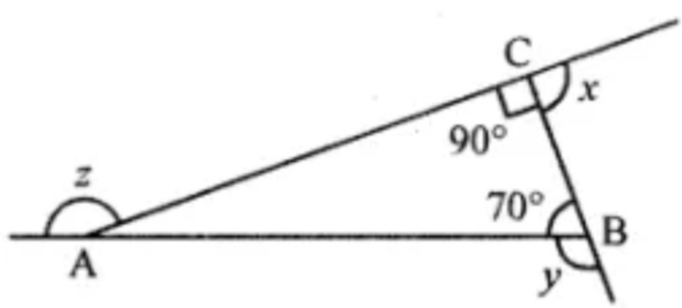


(ii) In the given figure, find x + y + z + w.



Solution:

(i) In ΔABC, Sides AB, BC, CA are produce in order in one side.



$\angle B = 70°, \angle C = 90°$

$\therefore \angle A = 180° - (\angle B + \angle C)$

= 180°– (70°+ 90°)

= 180°– 160° = 20°

But x + 90° = 180° (Linear pair)

∴ x = 180° – 90° = 90°

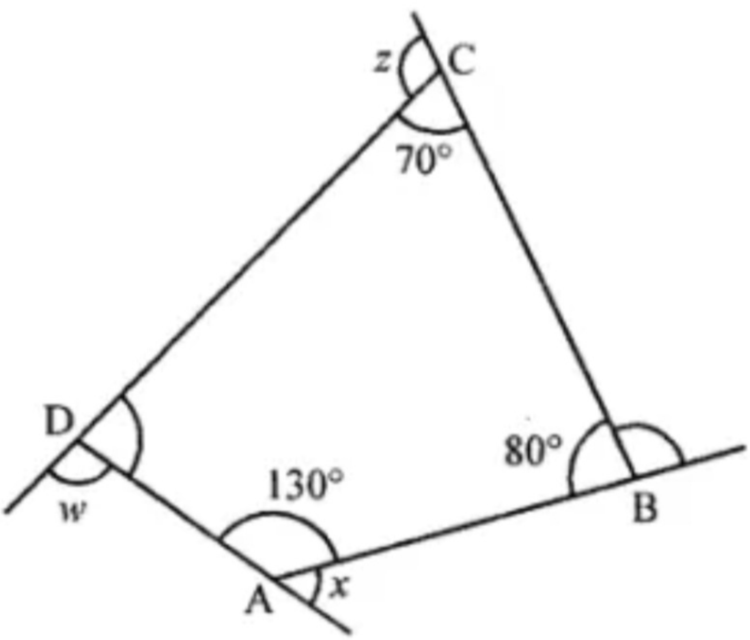
Similarly, y = 180° – 70° = 110°

y = 180° – 20°= 160°

x + y + z = 90° + 110°+ 160° = 360°

(ii) In the given figure,

ABCD is a quadrilateral whose sides are produced in order.



∠A = 130°, ∠B = 80°, ∠C = 70°

∴ ∠D = 360° – (∠A + ∠B + ∠C)

= 360° – (130° + 80° + 70°)

= 360° – 280° = 80°

Now, x + 130° = 180° (Linear pair)

∴ x = 180°– 130° = 50°

Similarly, y = 180° – 80° = 100°

z = 180° – 70°= 110°

w = 180°–80°= 100°

∴ x + y + z + w = 52° + 100° +110°+ 100°

= 360°

Question 14.

A heptagon has three equal angles each of 120° and four equal angles. Find the size of equal angles.

Solution:

Sum of angles of a heptagon = (2 × n – 4) × 90°

= (2 × 7 – 4) × 90°

= 10 × 90° = 900°

Sum of three angles are each equal i.e. 120°

= 120° × 3 = 360°

Sum of remaining 4 equal angles

= 900° – 360° = 540°

∴ Each angle = $\frac{540^\circ}{4}$ = 135°

Question 15.

The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 5. Find

(i) the measure of each exterior angle

(ii) the measure of each interior angle

(iii) the number of sides in the polygon.

Solution:

Ratio between an exterior and an interior angle = 1 : 5

Let exterior angle = x

Then interior angle = 5x

But sum of interior angle and exterior angle = 180°

∴ x + 5x = 180°

⇒ 6x = 180°

⇒ $\frac{180^\circ}{6}$ = 30°

(i) Measure of exterior angle x = 30°

(ii) and measure of interior angle = 5x = 5 × 30° = 150°

(iii) Let number of sides = n, then

$\frac{2n-4}{n}$ × 90° = 150°

$\Rightarrow \frac{2n-4}{n} = \frac{150°}{90} = \frac{5}{3}$

$\Rightarrow$ 6n − 12 = 5n

$\Rightarrow$ 6n − 5n = 12

$\Rightarrow$ n = 12

$\therefore$ Number of sides = 12

Question 16.

Each interior angle of a regular polygon is double of its exterior angle. Find the number of sides in the polygon.

Solution:

In a polygon,

Let exterior angle = x

Then interior angle = 2x

But sum of interior angle and exterior angle = 180°

$\therefore$ 2x + x = 180°

$\Rightarrow$ 3x = 180°

$\Rightarrow$ x = $\frac{180°}{3}$ = 60°

$\therefore$ Interior angle = 2 × 60° = 120°

Let number of sides of the polygon = x

Then $\frac{2n-4}{n}$ × 90° = 120°

$\Rightarrow \frac{2n-4}{n} = \frac{120°}{90} = \frac{4}{3}$

$\Rightarrow$ 6n − 12 = 4n

$\Rightarrow$ 6n − 4n = 12

$\Rightarrow$ 2n = 12

$\Rightarrow$ n = $\frac{12}{2}$ = 6

$\therefore$ Number of sides = 6