Question 1.

In the given figure, ABCD is a parallelogram.

Complete each statement along with the definition or property used.

(i) AD = ...

(ii) DC = ...

(iii) ∠DCB = ...

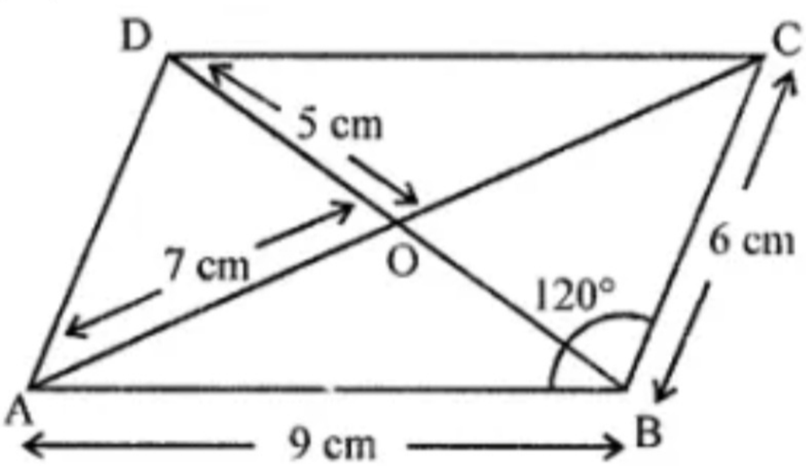
(iv) ∠ADC = ...

(v) ∠DAB = ...

(vi) OC = ...

(vii) OB = ...

(viii) m∠DAB + m∠CDA = ...



Solution:

In paralleloram ABCD

(i) AD = 6 cm (Opposite sides of parallelogram)

(ii) DC = 9 cm (Opposite sides of parallelogram)

(iii) ∠DCB = 60° (∵ ∠DCB + ∠CBA = 180°)

(iv) ∠ADC = ∠ABC = 120°

(v) ∠DAB = ∠DCB = 60°

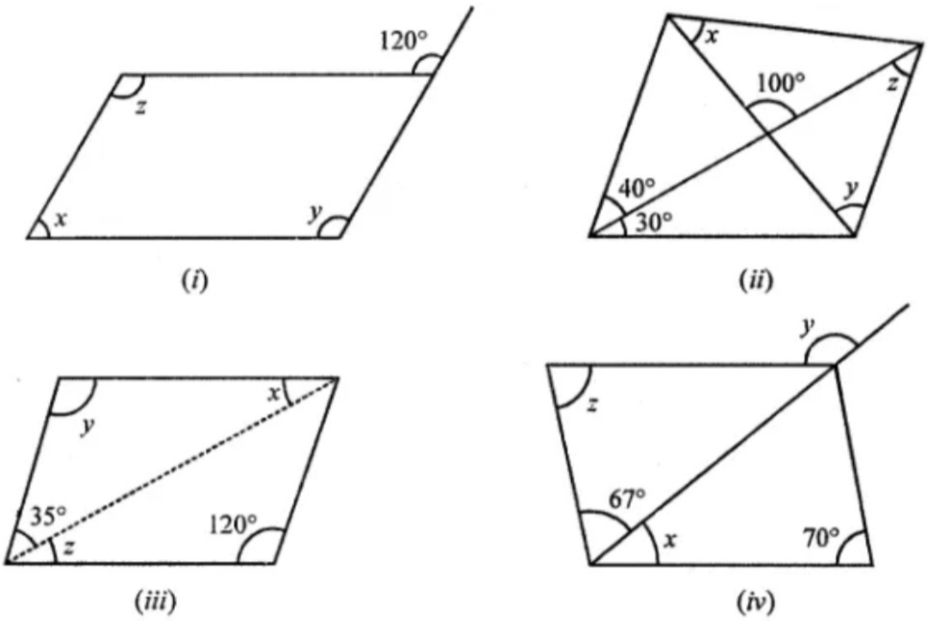
(vi) OC = AO = 7 cm

(vii) OB = OD = 5 cm

(viii) m∠DAB + m∠CDA = 180°

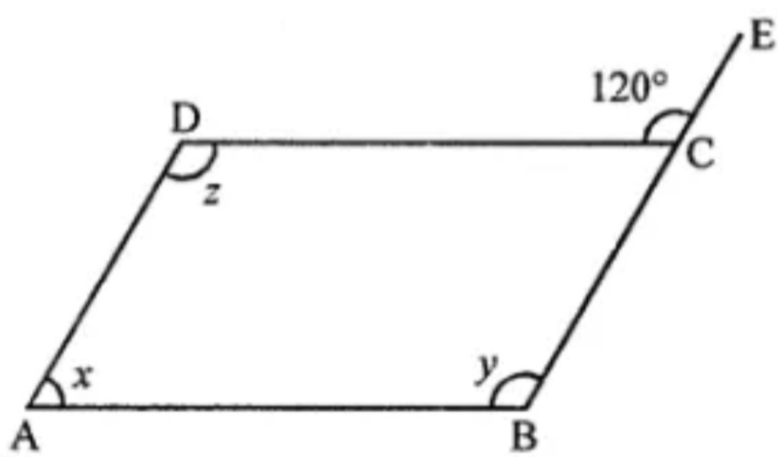
Question 2.

Consider the following parallelograms. Find the values of x, y, z in each.



Solution:

(i) ABCD is a parallelogram.



Side BC is produced to E

$\angle DCE = 120°$

But $\angle DCE + \angle DCB = 180°$ (Linear pair)

$\Rightarrow 120° + \angle DCB = 180°$

$\Rightarrow \angle DCB = 180° - 120° = 60°$

But $\angle A = \angle C$

$\Rightarrow x = 60°$

$\angle DCE = \angle ABC$ (Corresponding angles)

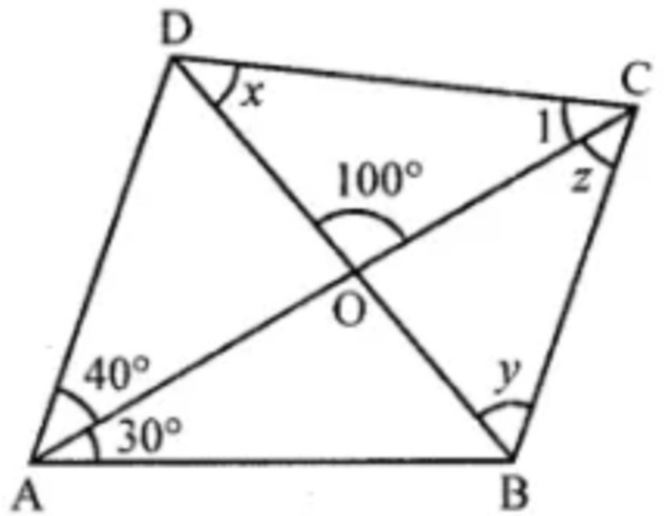
$\therefore y = 120°$

But z = y (Opposite angle of a ||gm)

$\Rightarrow z = 120°$

Hence $x = 60°$, $y = 120°$, $z° = 120°$

(ii) In parallelogram ABCD, diagonals bisect each other at O.



$\angle DAC = 40°$, $\angle CAB = 30°$, $\angle DOC = 100°$

$\angle ACB = \angle DAC = 40°$ (Alternate angles)

$\therefore z = 40°$

$\angle ACD = \angle CAB$ (A ltemate angles)

$\Rightarrow \angle ACD = 30°$

In ΔOCD,

$\angle DOC + \angle CDO + \angle OCD = 180°$ (Angles of a triangle)

$\Rightarrow 100° + x + 30° = 180°$

$\Rightarrow x + 130° = 180°$

$\Rightarrow x = 180° - 130° = 50°$

Ext. $\angle COD = y + z$

$100° = y + 40°$

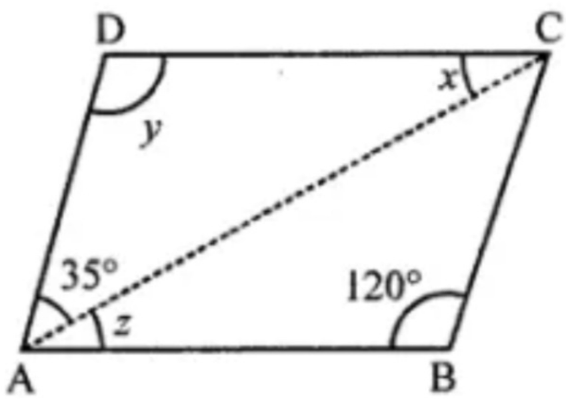
$\Rightarrow y = 100° - 40° = 60°$

$\therefore x = 50°$, $y = 60°$, $z = 40°$

(iii) In parallelogram ABCD, AC is its diagonal.

$\angle B = 120°, \angle DAC = 35°$

$\angle DAB + \angle ABC = 180°$ (Co–interior angles)



$35° + z + 120° = 180°$

$\Rightarrow 155° + z = 180°$

$\Rightarrow z = 180° - 155° = 25°$

But x = z (Alternate angles)

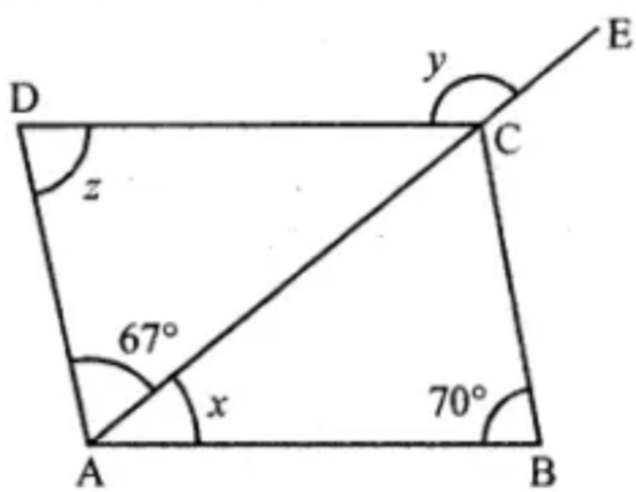
$\therefore x = 25°$

$y = \angle B$ (Opposite angles of a ||gm)

$y = 120°$

Hence $x = 25°, y = 120°, z = 25°$

(iv) In parallelogram ABCD



$\angle B = 70°, \angle DAC = 67°$

$\angle D = \angle B$ (Opposite angles of a ||gm)

$\Rightarrow z = 70°$

In ΔDAC

Ext. DCE = $\angle D + \angle DAC$

$y = z + 67°$

$y = 70° + 67° = 137°$

and $\angle DCA + \angle DCE = 180°$ (Linear pair)

$\angle DCA + 137° = 180°$

$\Rightarrow \angle DCA = 180° - 137° = 43°$

But $\angle CAB = \angle DCA$ (Alternate angles)

$\therefore x = 43°$

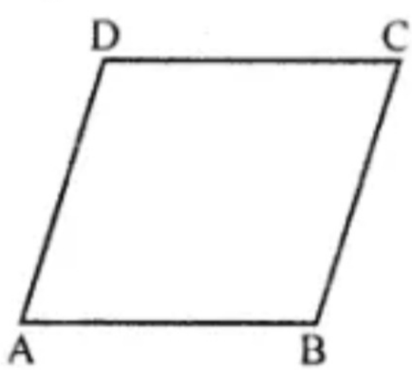
$\therefore x = 43°, y = 137°, z = 70°$

Question 3.

Two adjacent sides of a parallelogram are in the ratio 5 : 7. If the perimeter of parallelogram is 72 cm, find the length of its sides.

Solution:

In ||gm ABCD



AD : AB = 5 : 7

Perimeter of ||gm = 72 cm

$\Rightarrow 2(DA + AB) = 72$ cm

$\therefore DA + AB = \frac{72}{2} = 36$ cm

Let DA = 5x and AB = 7x

$5x + 7x = 36$

$\Rightarrow 12x = 36$

$\Rightarrow x = \frac{36}{12} = 3$

$\therefore AB = 7x = 7 \times 3 = 21$ cm

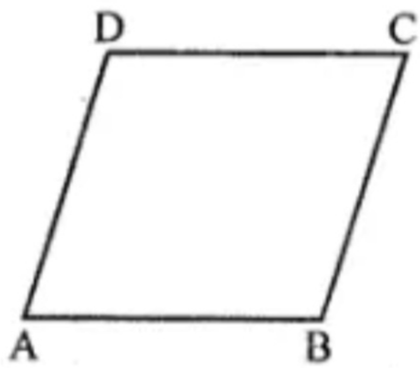
AD = 5x = 5 × 3 = 15 cm

Question 4.

The measure of two adjacent angles of a parallelogram are in the ratio 4 : 5. Find the measure of each angle of the parallelogram.

Solution:

In ||gm ABCD



$\angle A : \angle B = 4 : 5$

Let $\angle A = 4x$, $\angle B = 5x$

But $\angle A + \angle B = 180°$ (Cointerior angle)

$\therefore 4x + 5x = 180° \Rightarrow 9x = 180°$

$\Rightarrow x = \frac{180°}{9} = 20°$

$\therefore \angle A = 4x = 4 \times 20° = 80°$

$\angle B = 5x = 5 \times 20° = 100°$

But $\angle C = \angle A = 80°$ and $\angle D = \angle B = 100°$

(Opposite angles of a ||gm are equal)

Question 5.

Can a quadrilateral ABCD be a parallelogram, give reasons in support of your answer.

(i) $\angle A + \angle C = 180°$?

(ii) AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm?

(iii) $\angle B = 80°$, $\angle D = 70°$?

(iv) $\angle B + \angle C = 180°$?

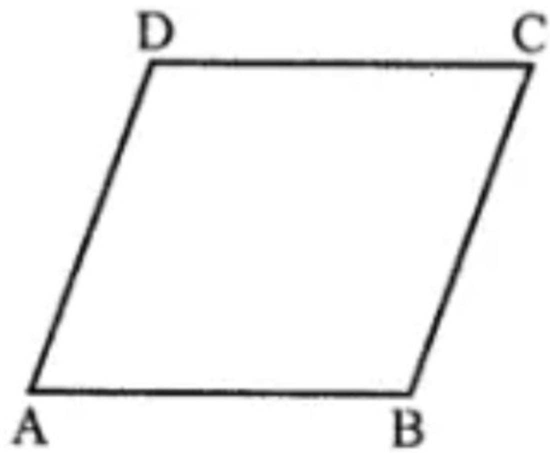
Solution:

Quadrilateral ABCD can be a parallelogram of opposite sides

are equal and opposite angles are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

and AB = DC, AD = BC



(i) $\angle A + \angle C = 180°$

It may be a parallelogram and may not be.

(ii) $\because$ AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm

$\because AB \neq DC$

(iii) $\angle B = 80°, \angle D = 70°$

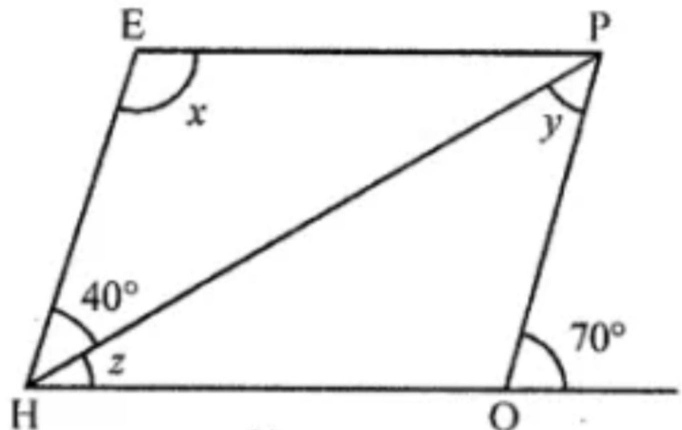
But there are opposite angles and $\angle B \neq \angle D$

(iv) $\therefore \angle B + \angle C = 180°$

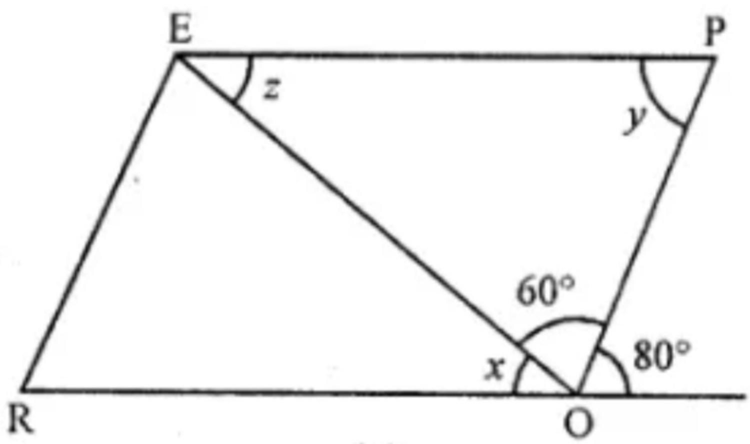
It may be or it may not be.

Question 6.

In the following figures HOPE and ROPE are parallelograms. Find the measures of angles x, y and z. State the properties you use to find them.



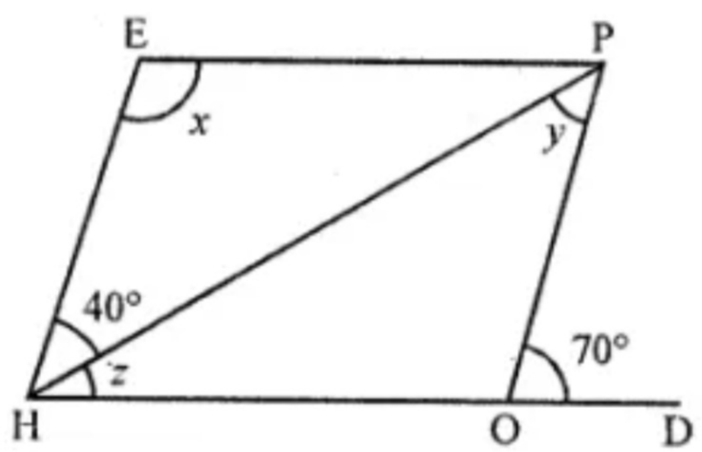
(i)



(ii)

Solution:

(i) In parallelogram HOPE, HO is produced to D



∠AOP + ∠POD = 180° (Linear pair)

∴ ∠AOP + 70°= 180°

∠AOP = 180°− 70°= 110°

But ∠AOP = ∠HEP (Opposite angles of a ||gm)

∠HEP = 110°

$\Rightarrow x = 110°$

$\angle HPO = \angle EHP$ (Alternate angles)

$\therefore y = 40°$

In ΔHOP, Ext. $\angle POD = y + z$

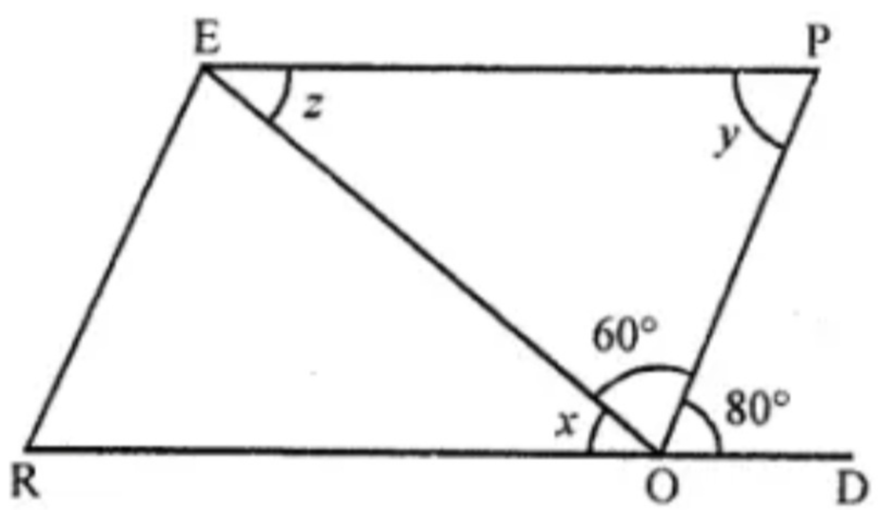
$\Rightarrow 70° = y + z$

$\Rightarrow 70° = 40° + z$

$\Rightarrow z = 70° - 40° = 30°$

$\therefore x = 110°, y = 40°, z = 30°$

(ii) In ||gm ROPE, RO is produced to D



$\angle POD = 80°, \angle EOP = 60°$

$\angle P = \angle POD$ (Alternate angles)

$\therefore y = 80°$

$\angle ROE + \angle EOP + \angle POD = 180°$ (Angles on one side of a line)

$x + 60° + 80° = 180° \Rightarrow x + 140° = 180°$

$\therefore x = 180° - 140° = 40°$

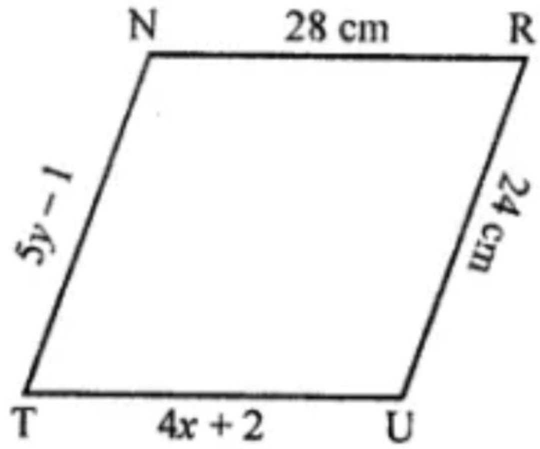
$z = x$ (Alternate angles)

$\therefore z = 40°$

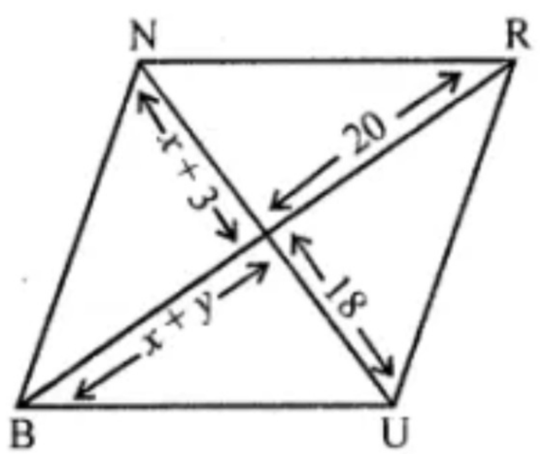
Hence, $x = 40°, y = 80°, z = 40°$

Question 7.

In the given figure TURN and BURN are parallelograms. Find the measures of x and y (lengths are in cm).



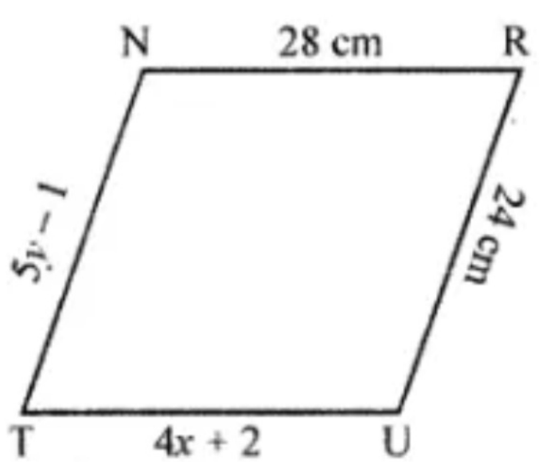
(i)



(ii)

Solution:

(i) We know that opposite sides of a parallelogram are equal.



∴ TU = RN

$4x + 2 = 28 \Rightarrow 4x = 28 - 2$

$\Rightarrow 4x = 26$

$\Rightarrow x = \frac{26}{4} = 6.5$ cm

and $5y - 1 = 24$

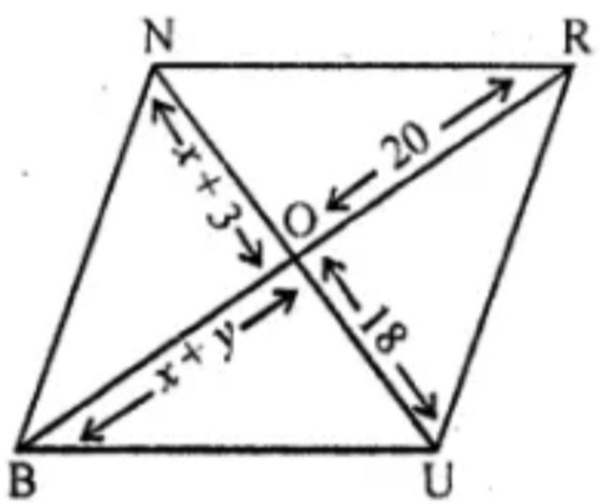
$\Rightarrow 5y = 24 + 1$

$\Rightarrow 5y = 25$

$\Rightarrow y = \frac{25}{5} = 5$

∴ x = 6.5 cm, y = 5 cm

(ii) We know that the diagonal of a parallelogram bisect each other.



∴ BO = OR

$\Rightarrow x + y = 20$ ....(i)

and UO = ON

$\Rightarrow x + 3 = 18$

$\Rightarrow x = 18 - 3 = 15$ From (i)

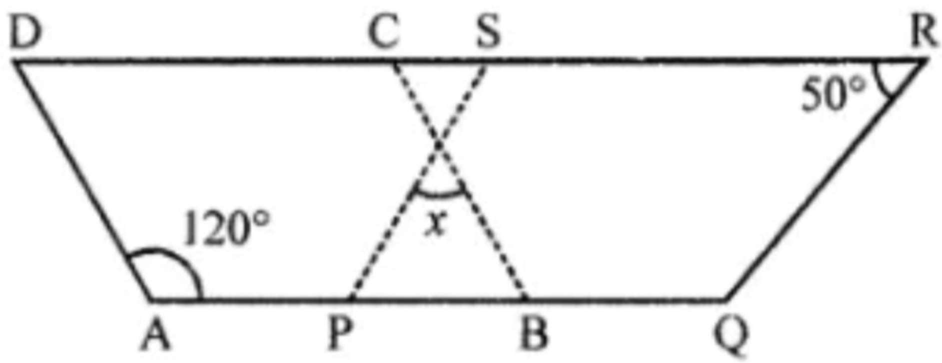
$15 + y = 20$

$\Rightarrow y = 20 - 15 = 5$

∴ x = 15, y = 5

Question 8.

In the following figure both ABCD and PQRS are parallelograms. Find the value of x.



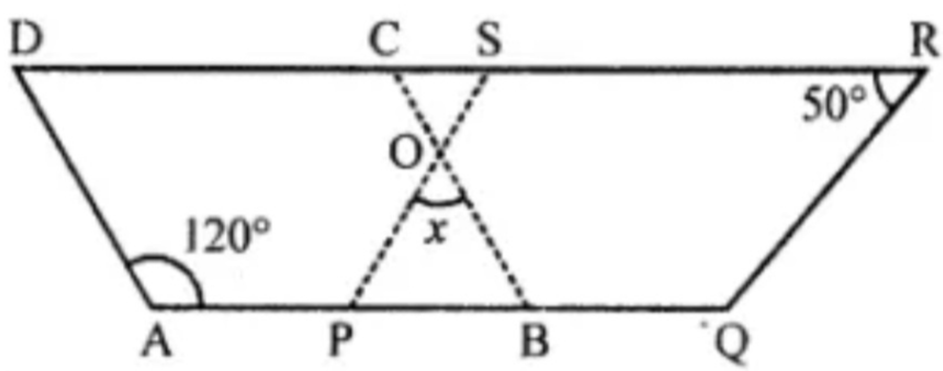
Solution:

Two parallelograms ABCD and PQRS in which

∠A = 120° and ∠R = 50°

∠A + ∠B = 180° (Co–interior angles)

120° + ∠B = 180°



⇒ ∠B = 180°– 120° = 60°

∠P = ∠R (Opposite angles of a ||gm)

∠P = 50°

Now in ΔOPB,

∠POB + ∠P + ∠B = 180° (Angles of a triangle)

x + 50° + 60° = 180°

x + 110° = 180° ⇒ x = 180°– 110° = 70°

∴ x = 70°

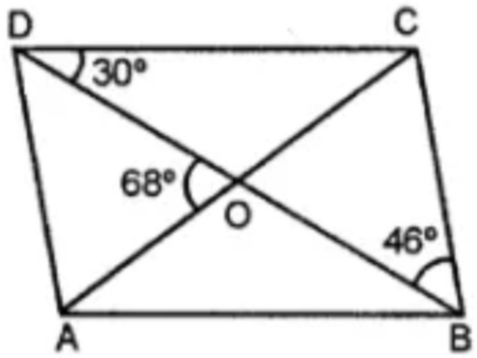
Question 9.

In the given figure, ABCD, is a parallelogram and diagonals intersect at O. Find :

(i) ∠CAD

(ii) ∠ACD

(iii) ∠ADC



Solution:

(i) ∠DBC = ∠BDA = 46° (alternate angles)

In Δ AOD,

46° + 68° + ∠CAD = 180° (∵ ∠CAD = ∠OAD)

∠CAD = 180°– 114° = 66°

(ii) ∠AOD + ∠COD = 180° (straight angle)

∴ ∠COD= 180°– 68°= 112°

In ΔCOD, 112° + 30° + ∠ACD = 180° (∵ ∠ACD = ∠OCD)

∠ACD = 180° – 112° – 30° = 38°

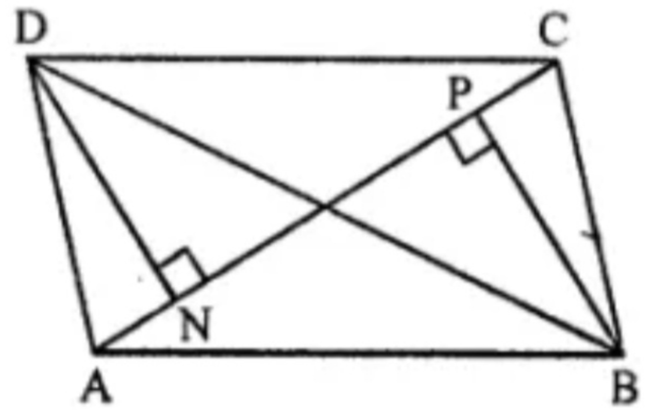
(iii) ∠ADC = 30° + 46° = 76° (∵ ∠ADC = ∠ADO + ∠ODC)

Question 10.

In the given figure, ABCD is a parallelogram. Perpendiculars DN and BP are drawn on diagonal AC. Prove that:

(i) ΔDCN ≅ ΔBAP

(ii) AN = CP



Solution:

In the given figure,

ABCD is a parallelogram AC is it's one diagonal.

BP and DN are perpendiculars on AC.

To prove :

(i) $\Delta DCN \cong \Delta BAP$

(ii) AN = CP

Proof: In ΔDCN and ΔBAP

DC=AB (Opposite sides of a ||gm)

$\angle N = \angle P$ (Each 90°)

$\angle DCN = \angle PAB$ (Alternate angle)

$\therefore \Delta DCN \cong \Delta BAP$ (AAS axiom)

$\therefore NC = AP$ (c.p.c.t.)

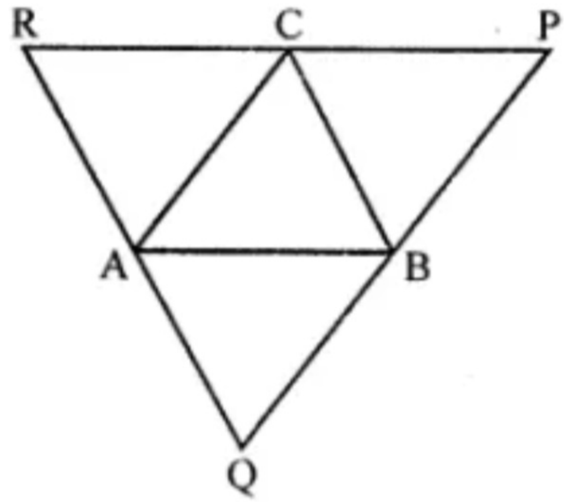
Subtracting NP from both sides.

$NC - NP = AP - NP$

$\therefore AN = CP$

Question 11.

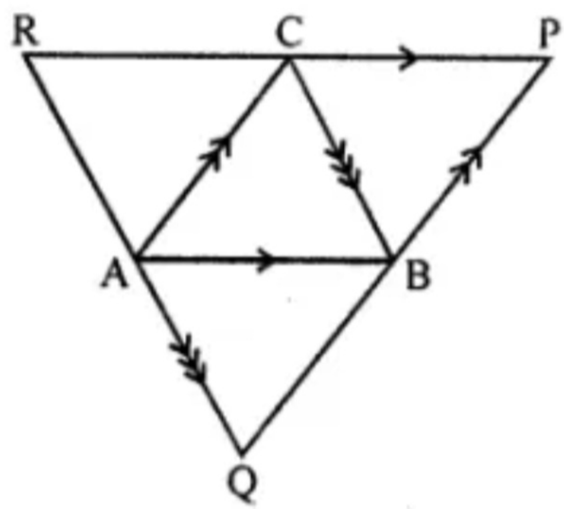
In the given figure, ABC is a triangle. Through A, B and C lines are drawn parallel to BC, CA and AB respectively, which forms a ΔPQR. Show that $2(AB + BC + CA) = PQ + QR + RP$.



Solution:

In the given figure, ABC is a triangle.

Through A, B and C lines are drawn parallel to BC, CA and AB respectively which forms ΔPQR.



To prove:

2(AB + BC + CA) = PQ + QR + RP

∵ BC || PR, AC || RQ

∴ ARBC is a ||gm

∴ AR = CB ....(i)

Similarly ABCP is a ||gm

∴ AP = BC ...(ii)

From (i) and (ii),

AR = AP or PR = 2BC ...(iii)

Similarly we can prove that

RQ = 2A and PQ = 2AB

Now perimeter of $\Delta PQR = PQ + QR + RP$

$= 2AB + 2AC + 2BC$

$= 2(AB + BC + CA)$

Hence $PQ + QR + RP = 2(AB + BC + CA)$