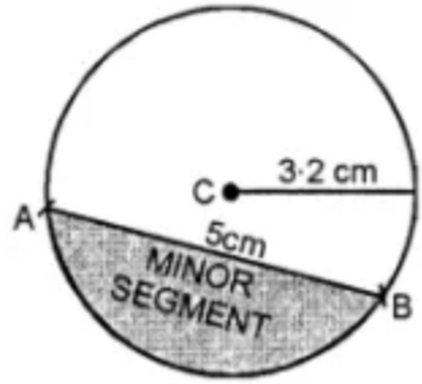
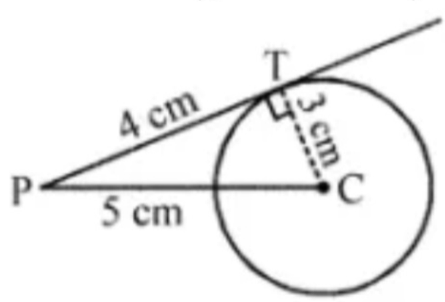


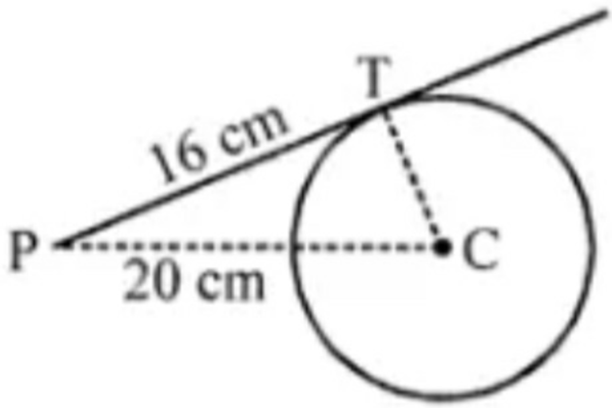
Question 3.
Find the length of the tangent drawn to a circle of radius 3 cm, from a point at a distance 5 cm from the centre.
Solution:
Draw a circle with centre C and radius CT = 3 cm.
Let PT be the tangent drawn from
point P to a circle with centre C.
CP = 5 cm
CT = 3 cm (given radius)



$\angle CTP = 90°$
$\because$ Radius is $\perp$ to tangent
From ΔCPT,
by Pythagoras theorem, we get
$CP^2 = PT^2 + CT^2$
$(5)^2 = PT^2 + 32$
$PT^2 = 25 - 9 = 16$
$PT = \sqrt{16} = 4$
Hence, length of tangent = 4 cm

Question 4.

In the adjoining figure, PT is a tangent to the circle with centre C. Given CP = 20 cm and PT = 16 cm, find the radius of the circle.



Solution:

We know that, radius is always ⊥ to longest.

i. e., CT ⊥ PT

∴ ΔCPT is right ∠d Δ

Where CP = hypotenuse

In rt. ΔCPT, by Pythagoras theorem,

$CP^2 = PT^2 + CT^2$

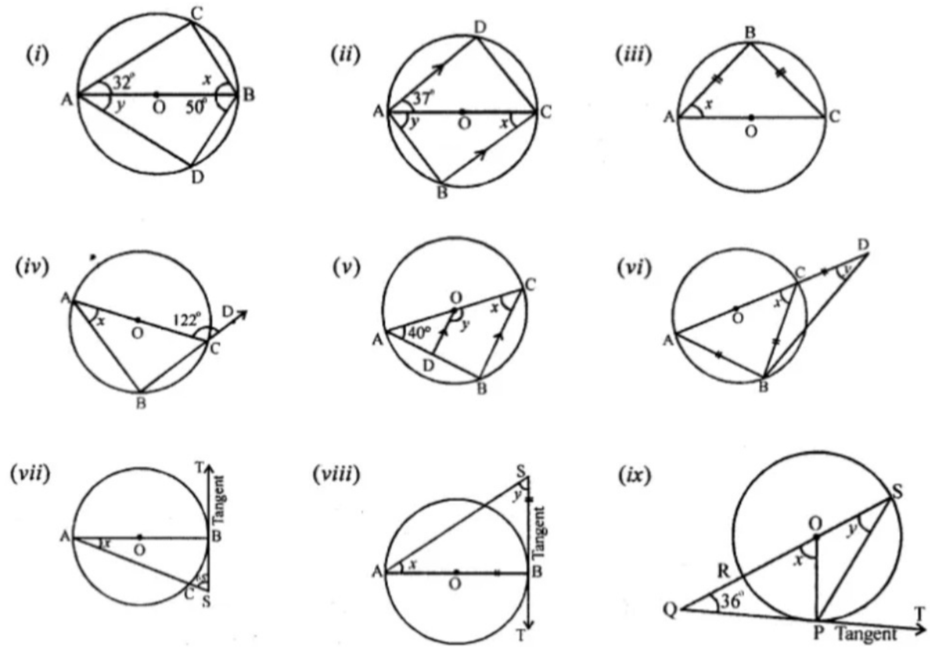
$CT^2 = CP^2 - PT^2 = 20^2 - 16^2 = 400 - 256 = 144$

$CT = \sqrt{144} = 12$ cm

Hence, radius of circle = 12 cm

Question 5.

In each of the following figure, O is the centre of the circle. Find the size of each lettered angle :



Solution:

(i) In the figure, AB is the diameter

and O is the centre of the circle ∠ CAB = 32°,

∠ ABD = 50° , ∠C = 90° (Angle in the semicircle)

By ∠ sum property of Δ

In ΔABC, ∠C + ∠CAB + ∠ABC = 180°

⇒ 90° + ∠CAB + x = 180°

⇒ 32° + x = 180° − 90°

⇒ x = 90° − 32°

⇒ x = 58°

Similarly in right Δ ADB

∠ADB = 90°

By ∠ sum property of Δ

∠ABD + ∠D + ∠BAD = 180°

⇒ 50° + 90° + ∠BAD = 180°

⇒ ∠y + 140° = 180°

⇒ ∠y = 180° − 140° = 40°

⇒ ∠y = 40°

(ii) In the figure,

AC in the diameter of circle with centre O

$\angle DAC = 37°$, $AD \parallel BC$

$\because AD \parallel BC$

$\angle ACB = \angle DAC$ (Alternate angles)

$\therefore x = 37°$

In ΔABC, $\angle B = 90°$ (Angle in a semicircle)

$\therefore$ By $\angle$sum property of Δ

$\angle x + \angle y + \angle B = 180°$

$\Rightarrow 37° + \angle y + 90° = 180°$

$\Rightarrow y = 180° - 127° = 53°$

(iii) In the figure,

AC is the diameter of the circle with centre O.

$BA = BC$

$\therefore \angle BAC = \angle BCA$ ($\angle$s of isosceles Δ)

But $\angle ABC = 90°$ (Angle in a semicircle)

In ΔABC

(By $\angle$ sum property of Δ)

$\angle BAC + \angle ABC + \angle BCA = 180°$

$\Rightarrow \angle BAC + \angle BCA = 180° - 90°$

$\Rightarrow x + x = 90°$

$\Rightarrow 2x = 90°$

$\therefore x = 45°$

(iv) In the figure,

AC is the diameter of the centre with centre O,

$\angle ACD = 122°$

$\because \angle ACB + \angle ACD = 180°$ (Linear pair)

$\Rightarrow \angle ACB + 122° = 180°$

$\Rightarrow \angle ACB + 180° - 122° = 58°$

(iv) In the figure,

AC is the diameter of the centre with centre O,

$\angle ACD = 122°$

$\because \angle ACB + \angle ACD = 180°$ (Linear pair)

$\Rightarrow \angle ACB + 122° = 180°$

$\Rightarrow \angle ACB + 180° - 122° = 58°$

In ΔABC, $\angle ABC = 90°$ (Angle in a semicircle)

$\therefore$ By angle sum prop, of Δ

$\angle ABC + \angle BCA + \angle ACB = 180°$

$90° + 58° + x = 180°$

$x = 180° - 148° = 32°$

(v) In the figures,

AC is the diameter of the circle with centre O,

$OD \parallel CB$ and $\angle CAB = 40°$

In ΔABC,

$\angle B = 90°$, (Angle in a semicircle)

By $\angle$ sum prop, of Δ

$\angle BCA + \angle ABC + \angle BAC = 180°$

$\angle BCA + \angle CAB + 90° = 180°$

$\therefore \angle BCA + \angle CAB = 90°$

$\Rightarrow x + 40° = 90° \Rightarrow x = 90° - 40° = 50°$

$\therefore x = 50°$

$\because OD \parallel CB$

$\therefore \angle AOD = \angle BCA$ (corresponding angles)

$\angle AOD = x = 50°$

But $\angle AOD + \angle DOC = 180°$ (Linear pair)

$\Rightarrow 50° + y = 180° \Rightarrow y = 180° - 50° = 130°$

Hence $x = 50°$ and $y = 130°$

(vi) In the figure,

AC is the diameter of the circle with centre O

BA = BC = CD

In ΔABC,

$\angle ABC = 90°$ (Angle in a semicircle)

By $\angle$ sum prop, of Δ

$\angle BAC + \angle BCA + \angle ABC = 180°$

$\angle BAC + \angle BCA + 90° = 180°$

$\therefore \angle BAC + \angle BCA = 90°$

But BA = BC (given)

$\therefore \angle BAC = \angle BCA = x$

$\therefore x + x = 90°$

$2x = 90°$

$\therefore x = 45°$

In ΔBCD,

BC = CD

$\therefore \angle CBD = \angle CDB = y$

and ext. $\angle ACB$ = Sum of interior opposite angles

$\angle CBD + \angle CDB$

$x = y + y = 2y$

$\therefore 2y = 45°$

$y = \frac{45°}{2} = 22.5°$ or $22\frac{1}{2}°$

(vii) In the figure,

AB is the diameter of circle with centre O.

ST is the tangent at B

$\angle ASB = 65°$

In ΔABS

$\because$ TS is the tangent and OB is the radius

$OB \perp ST$ or $\angle ABS = 90°$

But in ΔASB

$\angle BAC + \angle ASB + \angle ABS = 180°$ (Angles of a triangle)

$x + 65° + 90° = 180°$

$\Rightarrow x° + 155° = 180° \Rightarrow x = 180° - 155° = 25°$

Hence $x = 25°$

(viii)In the figure,

AB is the diameter of the circle with centre

O. ST is the tangent to the circle at B.

AB = BS

$\therefore$ ST is the tangent and OB is the radius

$\therefore OB \perp ST$ or $\angle OBS = 90°$

$\therefore$ In ΔABS,

$\angle BAS + \angle BSA + \angle ABS = 180°$

[By $\angle$ sum property of Δ]

$\Rightarrow \angle BAS + \angle BSA + 90° = 180°$

$\angle BAS + \angle BSA = 90° \Rightarrow x + y = 90°$

$\because AB = BS$

$\therefore x = y$

$\therefore x = y = \frac{90°}{2} = 45°$

(ix) In the figure,

RS is the diameter of the circle with centre O.

SR is produced to Q. QT is tangent to the circle at P

OP is joined.

$\angle Q = 36°$

QPT is tangent and OP is the radius of the circle

$\therefore$. $OP \perp QT$

$\angle OPQ = 90°$

$\therefore$ Now in ΔOPQ

By $\angle$ sum prop, of Δ

$\angle OQP + \angle POQ + \angle OPQ = 180°$

$\angle OQP + \angle POQ + 90° = 180°$

$\therefore \angle OQP + \angle POQ = 90°$

$\Rightarrow 36° + x = 90° \Rightarrow x = 90° - 36° = 54°$

In ΔOPS, $OP = OS$ (Radii of the circle)

$\therefore \angle OPS = \angle OSP = y$

and Ext. $\angle POQ = \angle OPS + \angle OSP$

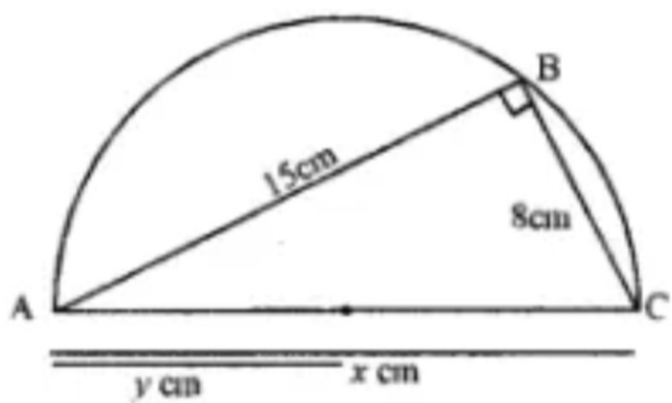
$= y + y = 2y$

$\Rightarrow 2y = x = 54°$

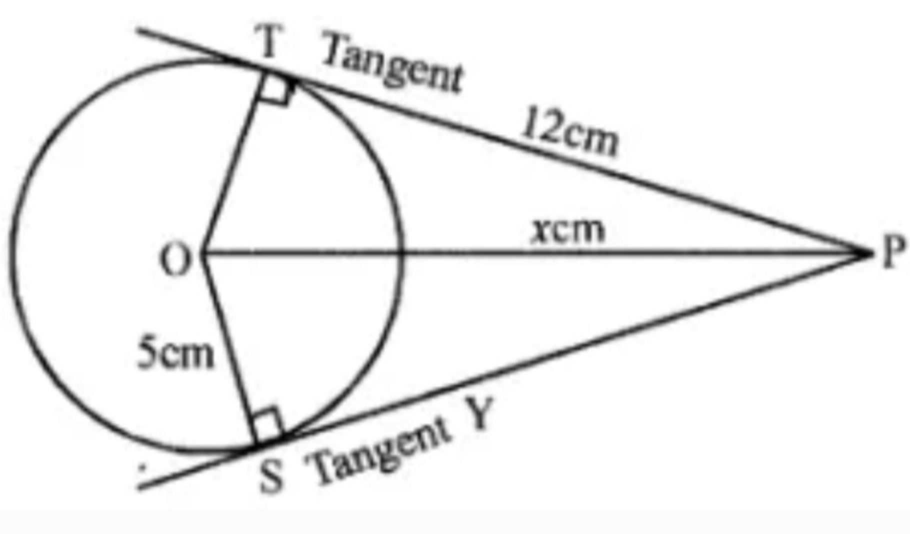
Question 6.

In each of the following figures, O is the j centre of the circle. Find the values of x and y.

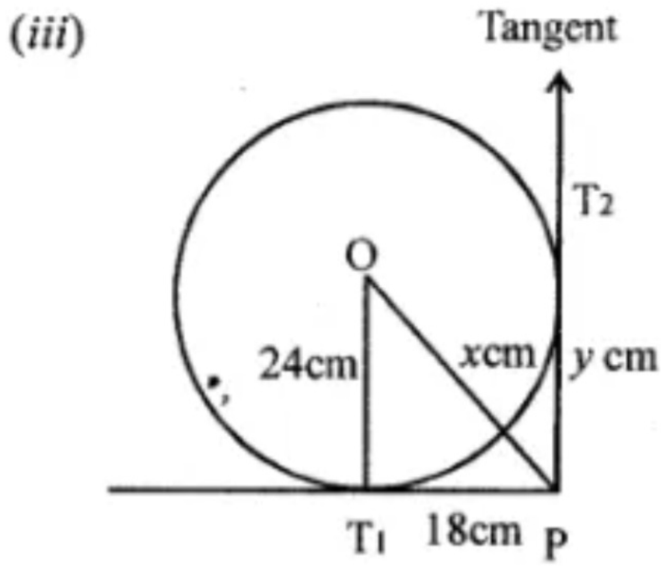
(i)



(ii)



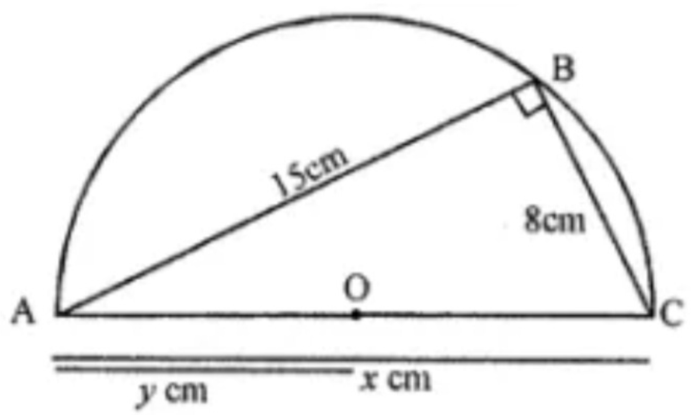
(*iii*)



Solution:

(i) O is the centre of the circle

AB = 15 cm, BC = 8cm



∠ABC = 90° (Angles in a semicircle)

By Pythagoras Theorem,

$AC^2 = AB^2 + AC^2$

$= (15)^2 + (8)^2 = 225 + 64$

$= 289 = (17)^2$

∴ AC = 17 cm

∴ x = 17 cm

∴ $y = \frac{1}{2}$

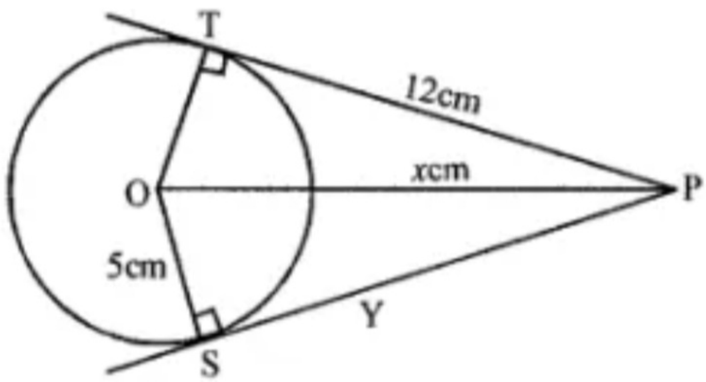
(∵ AC is the diameter and AO is the radius of the circle)

$= \frac{1}{2} \times 17 = \frac{17}{2}$ cm = 8.5 cm

(ii) O is the centre of the circle.

PT and PS are the tangents to the circle from P.

OS and OT are the radii of the circle



$\therefore \angle OSP = \angle OTP = 90°$

OS = OT = 5 cm, PT = PS = 12 cm

Now in right ΔOPT (By Pythagoras Theorem)

$OP^2 = OT^2 + PT^2 = (5)^2 + (12)^2$

$= 25 + 144 = 169 = (13)^2$

$\therefore OP = 13 \text{ cm} \Rightarrow x = 13\text{cm}$

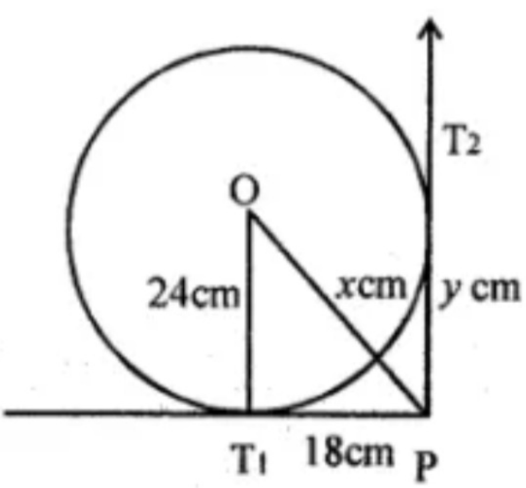
$\because PS = PT = 12 \text{ cm}$

$\therefore y = 12 \text{ cm}$

(iii) O is the centre of the circle OT_1 is the radius,

From P, PT_1 and PT_2 are the tangents.

$OT_1 = 24$ cm $PT_1 = 18$cm



$\because$ OT, is the radius and PT_1 is the tangent

$\therefore OT_1 \perp PT_1$

Now in right ΔOPT, (By Pythagoras Theorem)

$OP^2 = OT_1^2 + PT_1^2 = (24)^2 + (18)^2$

$= 576 + 324 = 900 = (30)^2$

$\therefore OP = 30$

$\Rightarrow x = 30$ cm

$\because PT_1$ and PT_2 are the tangents from P

$\therefore PT_2 = PT_1 = 18$ cm

$\Rightarrow y = 18$ cm