Question 1.

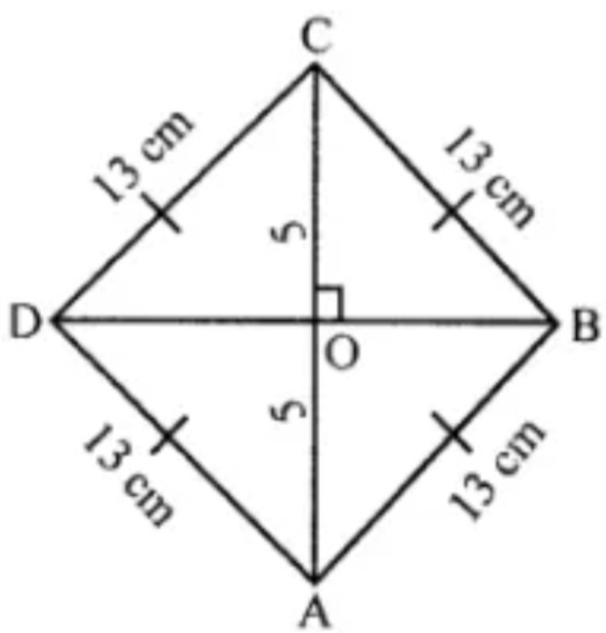
Each sides of a rhombus is 13 cm and one diagonal is 10 cm. Find

(i) the length of its other diagonal

(ii) the area of the rhombus

Solution:

(i) Side of rhombus = 13 cm.



Length of diagonal AC = 10 cm.

∴ OC = 5 cm.

Since the diagonals of rhombus are ± to each other

∴ ΔBOC is rt. angled.

Hence, $BC^2 = OC^2 + OB^2$

$13^2 = 5^2 + OB^2$

$\Rightarrow OB^2 = 169 - 25 = 144$

$\Rightarrow OB = \sqrt{144} = \sqrt{12 \times 12} = 12$ cm

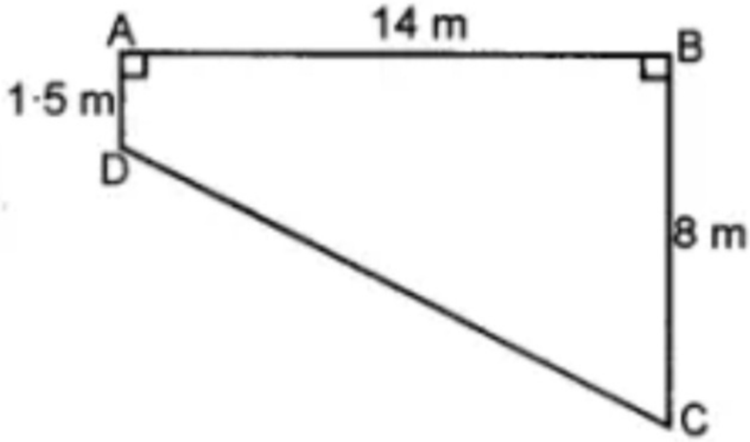
∴ Diagonal BD = 2 × OB = 2 × 12 = 24 cm

(ii) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 10 \times 24 = 120 cm^2$

Question 2.

The cross–section ABCD of a swimming pool is a trapezium. Its width AB = 14 m, depth at the shallow end is 1–5 m and at the deep end is 8 m. Find the area of the cross–section.



Solution:

Here, two parallel sides of trapezium are AD and BC and distance between them is 14 m.

$\therefore$ Area of trapezium = $\frac{1}{2}(1\cdot5 + 8) \times 14$

$= \frac{1}{2} \times 9\cdot5 \times 14 = 66 \times 5 \text{ m}^2$

Question 3.

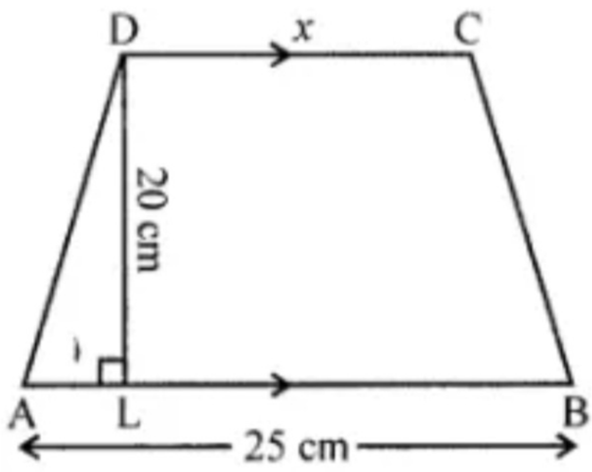
The area of a trapezium is 360 m^2, the distance between two parallel sides is 20 m and one of the parallel side is 25 m. Find the other parallel side.

Solution:

Area of a trapezium = 360 m^2

Distance between two parallel lines = 20 m

One parallel side = 25 m



Let Second parallel side = 11 m

$\therefore$ Area = $\frac{1}{2}(25 + x) \times 20$

$\Rightarrow 360 = \frac{1}{2}(25 + x) \times 20$

$\therefore x = 36 - 25 = 11$ m

$\therefore$ Second parallel side = 11 m

Question 4.

Find the area of a rhombus whose side is 6.5 cm and altitude is 5 cm. If one of its diagonal is 13 cm long, find the length of other diagonal.

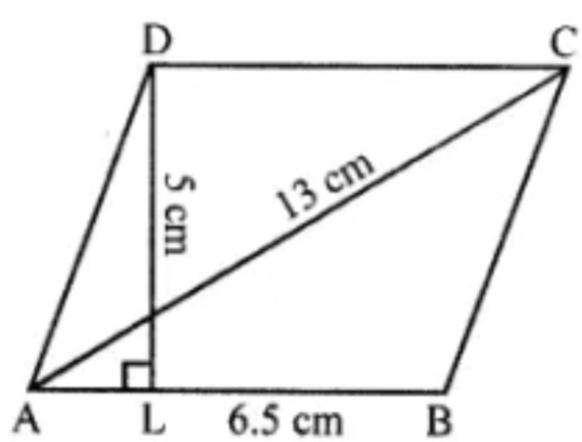
Solution:

Side of rhombus = 6.5 cm

and altitude = 5 cm

Area of a rhombus = Side × Altitude = 6.5 × 5 = 32.5 cm^2

One diagonal = 13 cm



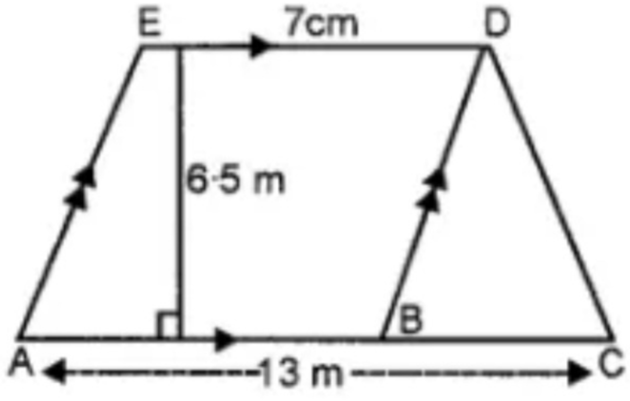
Question 5.

From the given diagram, calculate

(i) the area of trapezium ACDE

(ii) the area of parallelogram ABDE

(iii) the area of triangle BCD.



Solution:

(i) Area of trapezium ACDE

$= \frac{1}{2}(AC + DE) \times h$

$= \frac{1}{2}(13 + 7) \times 6.5 = 65\ m^2$

(ii) Area of parallelogram ABDE = b × h = $\frac{1}{2} \times 6 \times 6{\cdot}5$ = $15{\cdot}5\ m^2$

[∵ BC = AC − AB = 13 − 7 = 6 m]

Question 6.

The area of a rhombus is equal to the area of a triangle whose base and the corresponding altitude are 24.8 cm and 16.5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.

Solution:

Base of triangle = 24.8 cm

and altitude = 16.5 cm

Area = $\frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 24.8 \times 16.5 \text{ cm}^2 = 204.6 \text{ cm}^2$

Now, Area of Δ = Area of rhombus

∴ Area of rhombus = 204.6 cm^2

Length of one diagonal = 22 cm

Area of rhombus = $\frac{1}{2}$(First diagonal × Second diagonal)

∴ Second diagonal = $\frac{\text{Area } \times 2}{\text{First diagonal}}$

$= \frac{204.6 \times 2}{22} = 18.6$ cm

Question 7.

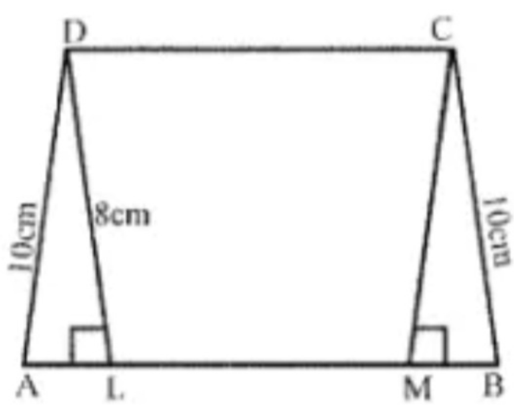
The perimeter of a trapezium is 52 cm. If its non–parallel sides are 10 cm each and its altitude is 8 cm, find the area of the trapezium.

Solution:

Perimeter of a trapezium = 52 cm

Length of each non–parallel side = 10 cm

Altitude DL = 8 cm



In right ΔDAL (By Pythagoras Theorem)

$DA^2 = DL^2 + AL^2$

$\Rightarrow (10)^2 = (8)^2 + AL^2$

$\Rightarrow 100 = 64 + AL^2$

$\Rightarrow AL^2 = 100 - 64 = 36 = (6)^2$

∴ AL = 6 cm

Similarly BM = 6 cm

and DC = LM

Also, perimeter = AB + BC + CD + DA

and CD = DA

∴ CD + DA = 2DA

But AB + CD = Perimeter − 2 AD

= 52 − 2 × 10 = 52 − 20 = 32 cm

Now area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × altitude

= $\frac{1}{2}$ × 32 × 8 = 128 cm^2

Question 8.

The area of a trapezium is 540 cm^2. If the ratio of parallel sides is 7 : 5 and the distance between them is 18 cm, find the lengths of parallel sides.

Solution:

Let, the two parallel sides of trapezium are 7x and 5x.

Height = 18 cm

⇒ Area of trapezium = $\frac{1}{2}$ [(Sum of ||gm sides) × height]

540 = $\frac{1}{2}$(7x + 5x) × 18

∴ 540 = $\frac{1}{2}$ × 12x × 18

or 108x = 540

⇒ x = 5 cm

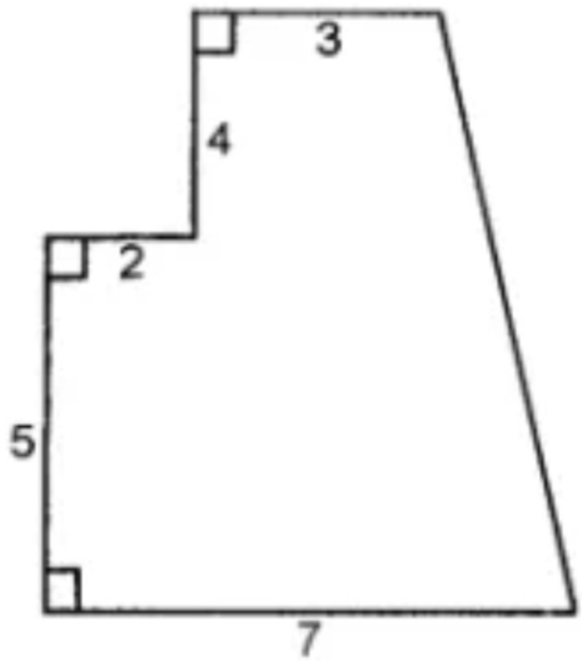
Hence, two parallel sides are

7x = 7 × 5 = 35 cm

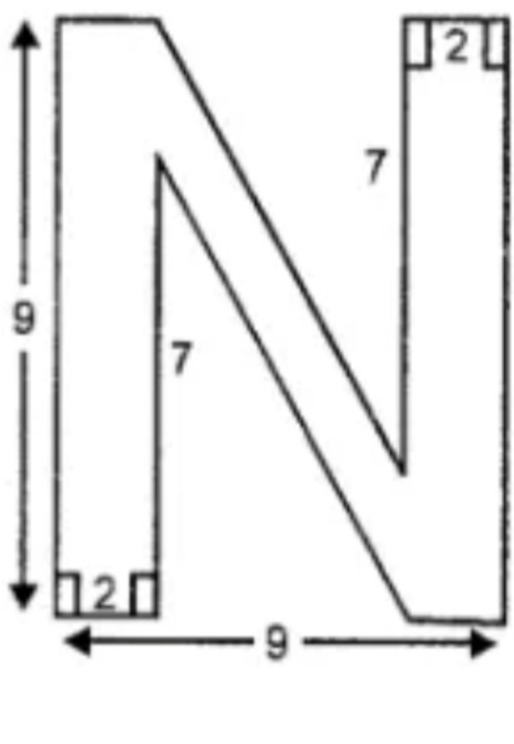
and 5x = 5 × 5 = 25 cm

Question 9.

Calculate the area enclosed by the given shapes. All measurements are in cm.



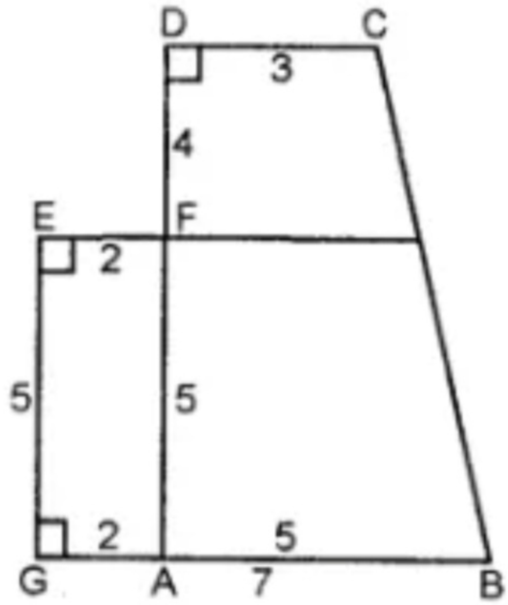
(*i*)



(*ii*)

Solution:

(i) Area of trapezium ABCD



$= \frac{1}{2}$ (Sum of opposite ||gm sides) × height

$= \frac{1}{2}$ [(AB + CD) × (AF + FD)]

$= \frac{1}{2}$[(AB + CD) × (AF + FD)

$= \frac{1}{2}$[(5 + 3) × (5 + 4)]

$= \frac{1}{2}$ (5 + 3) × 9 = 36 cm^2

Area of rectangle GAFE = Length × Breadth

= 2 × 5 = 10 cm^2

Total area of the figure

= Area of trapezium ABCD + Area of rectangle GAFE

= (36 + 10) cm^2

= 46 cm^2

(iii) Area of rectangle ABCD

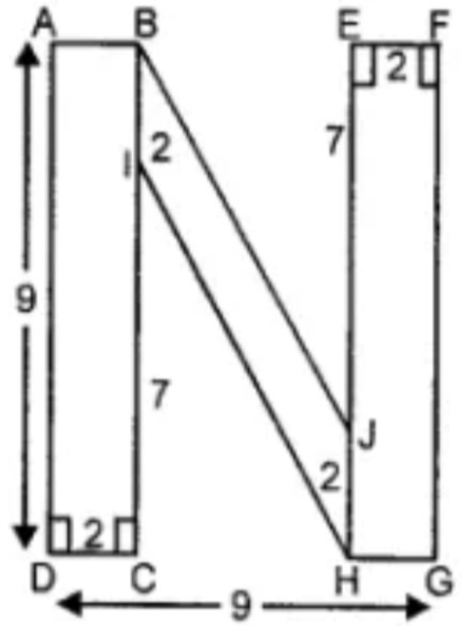
Area of given figure = Area of rect. ABCD

+ Area of ||gm BIHJ + Area of rectangle EFGH.

= Length × Breadth

= AD × DC

= 9 × 2 = 18 cm^2



Area of rectangle EFGH = Length × Breadth

= (EJ + JH) × EF

= (7 + 2) × 2

= 9 × 2 = 18 cm^2

Area of parallelogram BIHJ = 2 × 5 = 10 cm^2

[∵ Distance between BI and HJ = 9 − 2 − 2 = 5 cm]

Total area of the figure = (18 + 18 + 10) cm^2 = 46 cm^2

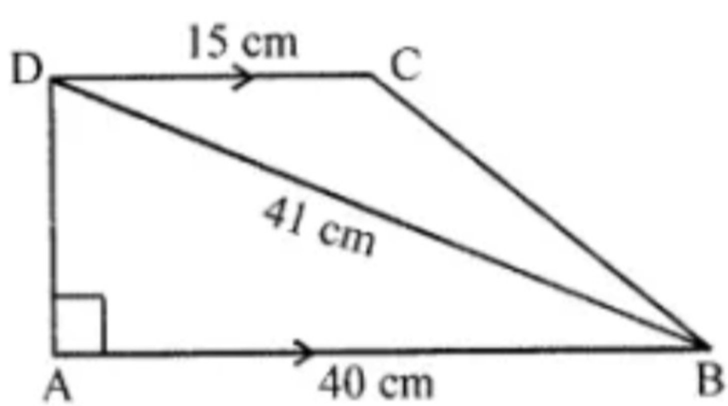
Question 10.

From the adjoining sketch, calculate

(i) the length AD

(ii) the area of trapezium ABCD

(iii) the area of triangle BCD



Solution:

(i) In right angled Δ ABD (By Pythagoras Theorem)

$BD^2 = AD^2 + AB^2$

$\Rightarrow AD^2 = BD^2 - AB^2 = (41)^2 - (40)^2 = 1681 - 1600 = 81$

$\therefore AD = \sqrt{81} = 9$ cm.

(ii) Area of trapezium ABCD

$= \frac{1}{2}$ (Sum of opposite ||gm lines) × height

$= \frac{1}{2}(AB + CD) \times AD$

$= \frac{1}{2}(40 + 15) \times 9 = 247 \cdot 5\ cm^2$

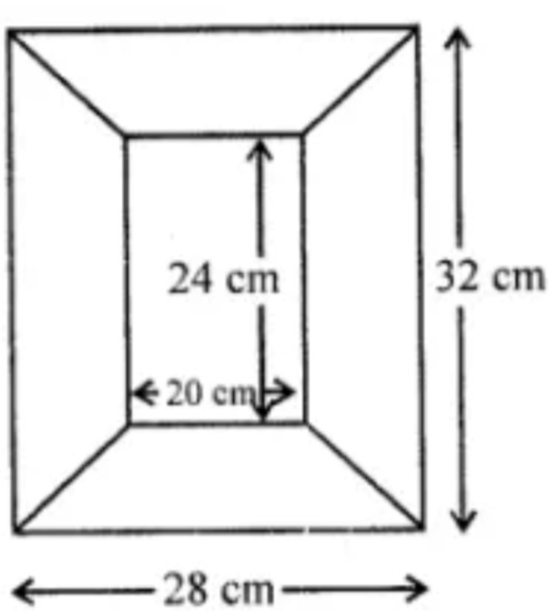
(iii) Area of triangle BCD = Area of trapezium ABCD – Area of Δ ABD

$= (247 \cdot 5 - \frac{1}{2} \times 40 \times 9)\ cm^2$

$= (247 \cdot 5 - 180)\ cm^2 = 67 \cdot 5\ cm^2$

Question 11.

Diagram of the adjacent picture frame has outer dimensions = 28 cm × 32 cm and inner dimensions 20 cm × 24 cm. Find the area of each section of the frame, if the width of each section is same.



Solution:

Outer length of the frame = 32 cm

and outer breadth = 28 cm

Inner length = 24 cm

and outer breadth = 20 cm

∴ Width of the frame = $\frac{32-24}{2} = \frac{8}{2}$ = 4 cm

i. e., height = 4 cm

Now area of each portion of length side

= $\frac{1}{2}$ (24 + 32) × 4

= $\frac{1}{2}$ × 56 × 4 = 112 cm^2

and area of each portion of breadth side

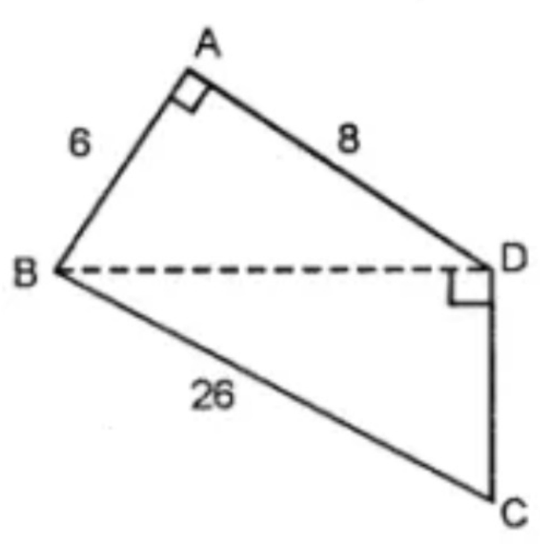
= $\frac{1}{2}$(20 + 28) × 4

= $\frac{1}{2}$ × 48 × 4 = 96 cm^2

∴ Area each sections =112 cm^2, 96 cm^2, 112 cm^2, 96 cm^2

Question 12.

In the given quadrilateral ABCD, ∠BAD = 90° and ∠BDC = 90°. All measurements are in centimetres. Find the area of the quadrilateral ABCD.



Solution:

In right angled triangle ABD, (By Pythagoras Theorem)

$BD^2 = AB^2 + AD^2 = (6)^2 + (8)^2 = 36 + 64 = 100 \text{ cm}^2$

$BD = \sqrt{100\text{cm}^2}$

$\therefore$ BD = 10 cm.

Now, Area of A ABD = $\frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 8 = 24\text{cm}^2$ ...(i)

In Δ BDC, BD = 10 cm.,

BC = 26 cm., DC = ?

By Pythagoras theorem,

$BC^2 = BD^2 + DC^2$

$(26)^2 = (10)^2 + DC^2$

$676 - 100 = DC^2$

$\Rightarrow DC = \sqrt{576} = 24$ cm.

Now, Area of Δ BDC = $\frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 24 \times 10 = 12 \text{ cm}^2$ ...(ii)

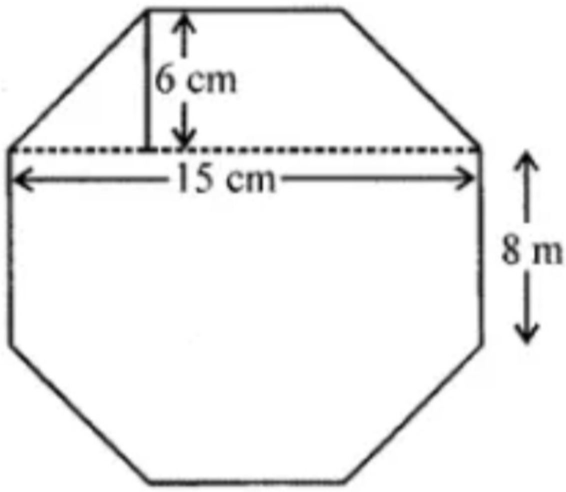
Add (i) and (ii), we get

Area of ΔABD + Area of ΔBDC = $(24 + 120) \text{ cm}^2$

Area of quadrilateral ABCD = 144 cm^2

Question 13.

Top surface of a raised platform is in the shape of a regular octagon as shown in the given figure. Find the area of the octagonal surface.

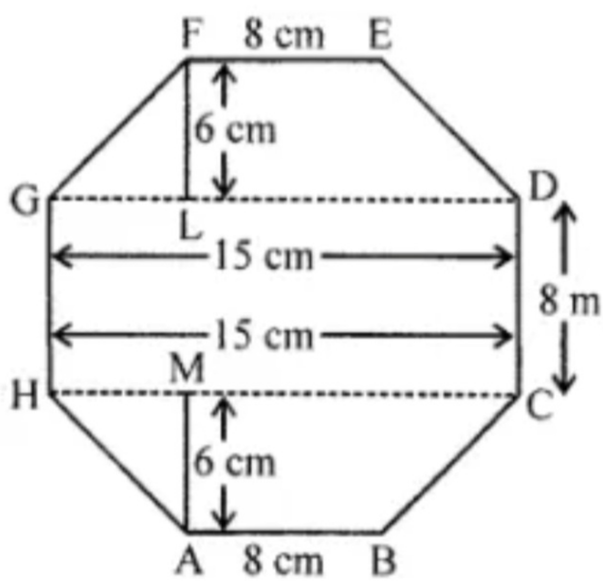


Solution:

Raised surface of platform is in the shape of regular octagon ABCDEFGH.

Each side = 8 cm, join HC

GD = HC = 15 cm, FL = AM = 6 cm



Now in each trapezium parallel sides are 15 cm and 6 cm

and height = 6 cm

∴ Area of each trapezium FEDG

$= \frac{1}{2}(GD + FE) \times FL$

$= \frac{1}{2}(15 + 8) \times 6$

$= 23 \times 3 \text{ cm}^2 = 69 \text{ cm}^2$

Area of trapezium FEDG = Area of trapezium ABCH = 69 cm^2

Area of trapezium FEDG = Area of trapezium ABCH = 69 cm^2

and area of rectangle HCDG

= HC × CD = 15 × 8 = 120 cm^2

Total area = Area of trapezium FEDG + Area of trapezium ABCH

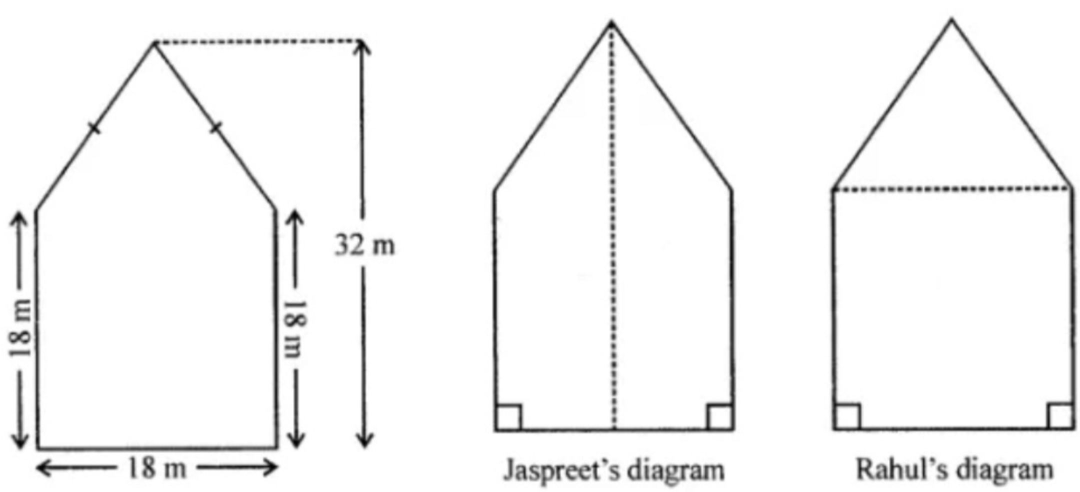
+ Area of rectangle HCDG.

Total area = 69 + 69 + 120 = 258 cm^2

Question 14.

There is a pentagonal shaped park as shown in the following figure:

For finding its area Jaspreet and Rahul divided it in two different ways.

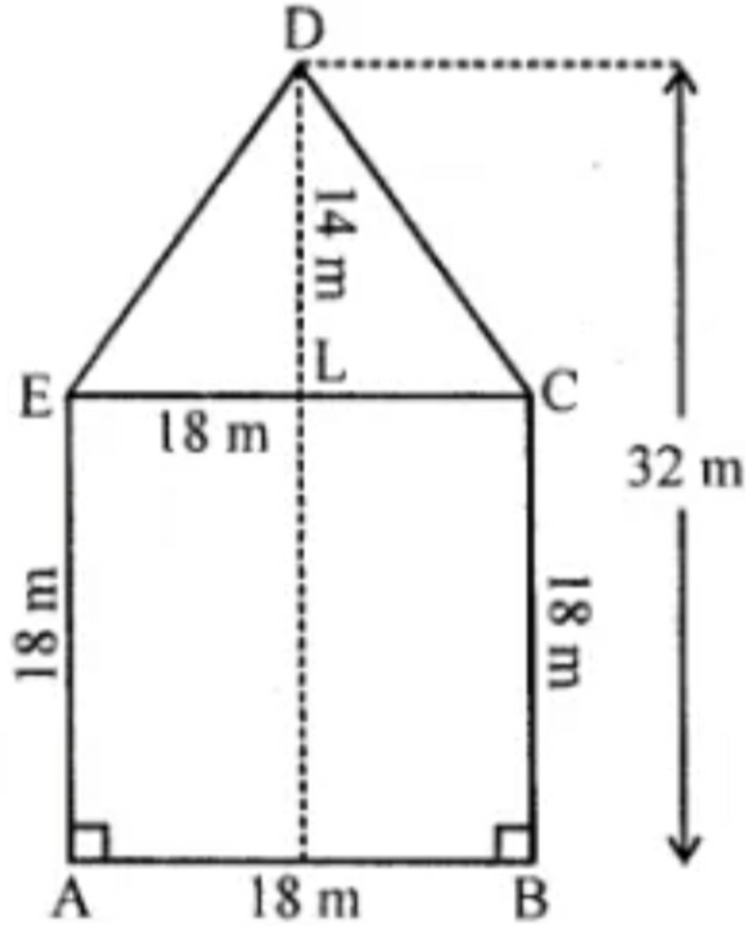


Find the area of this park using both ways. Can you suggest some other way of finding its area?

Solution:

The pentagonal shaped park is shown in the given figure,

in which DL ⊥ CE and is produced to M.



∴. DM = 32 m

LM = CB = 18 m

∴ DL = 32 − 18 = 14 m

(i) According to Jaspreet's the figure is divided into two equal trapezium in area: DEAM and DCBM

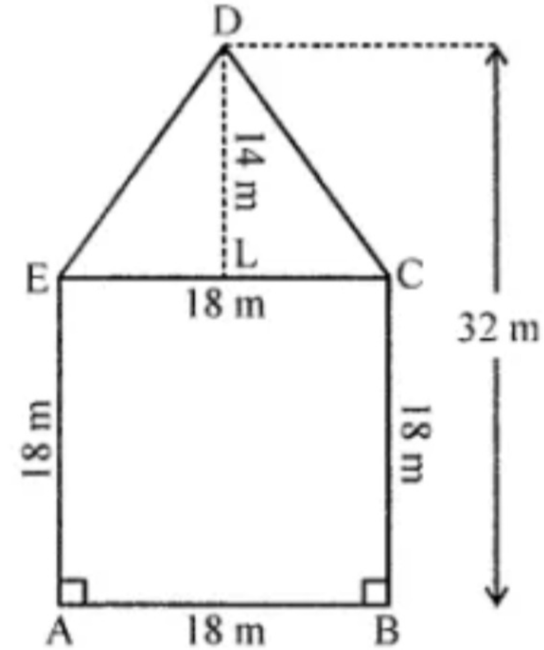
Area of trapezium DEAM

$= \frac{1}{2}(AE + DM) \times AM$

$= \frac{1}{2}(32 + 18) \times 9$

$= \frac{50 \times 9}{2} = 225 m^2$

According to Rahul's the figure is divided into shapes one square and on isosceles triangle.



Area of square ABCE = $(\text{Side})^2 = (18)^2 = 324\ m^2$

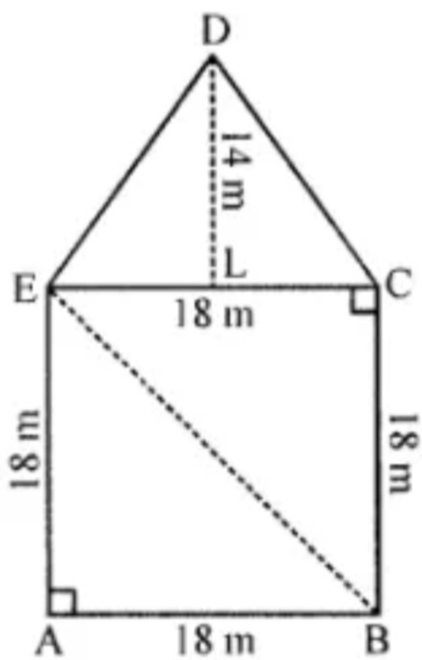
and area of isosceles ΔEDC

$= \frac{1}{2} \times EC \times DC$

$= \frac{1}{2} \times 18 \times 14 = 126\ m^2$

$\therefore$ Total area = $225 \times 2 = 450\ m^2$

Third way to find out the area of given figure is as follow:



Here, $DL \perp ED$, DL = 14 m

Area of $\Delta DEC = \frac{1}{2} \times EC \times LD$

$= \frac{1}{2} \times 18 \times 14 = 126\ m^2$

Area of $\Delta AEB = \frac{1}{2} \times AB \times AE$

$= \frac{1}{2} \times 18 \times 18 = 162\ m^2$

Area of $\Delta BEC = \frac{1}{2} \times BC \times EC$

$= \frac{1}{2} \times 18 \times 18 = 162\ m^2$

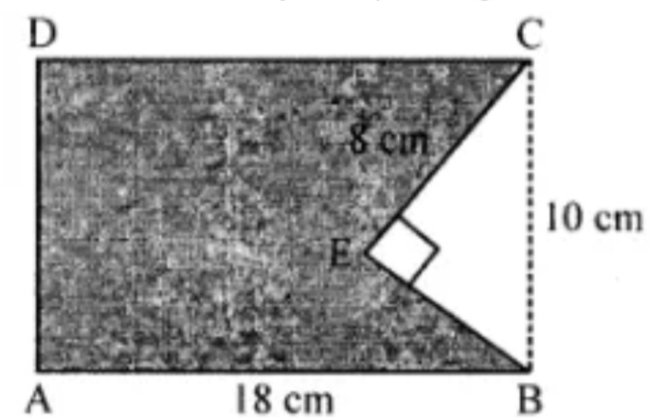
Now, area of pentagon ABCDE = Area ΔDEC

+ Area of ΔAEB + Area of ΔBEC

$= (126 + 162 + 162)\ m^2 = 450\ m^2$

Question 15.

In the diagram, ABCD is a rectangle of size 18 cm by 10 cm. In Δ BEC, $\angle E = 90°$ and EC = 8 cm. Find the area enclosed by the pentagon ABECD.



Solution:

Area of rectangle ABCD = Length × Breadth

$= 18 \times 10 = 180\ cm^2$

In right angled Δ BEC,

$BC^2 = CE^2 + BE^2$ (By Pythagoras theorem)

$(10)^2 = 8^2 + BE^2$

$\therefore BE^2 = 100 - 64 = 36$

$\Rightarrow BE = \sqrt{36} \Rightarrow BE = 6$ cm.

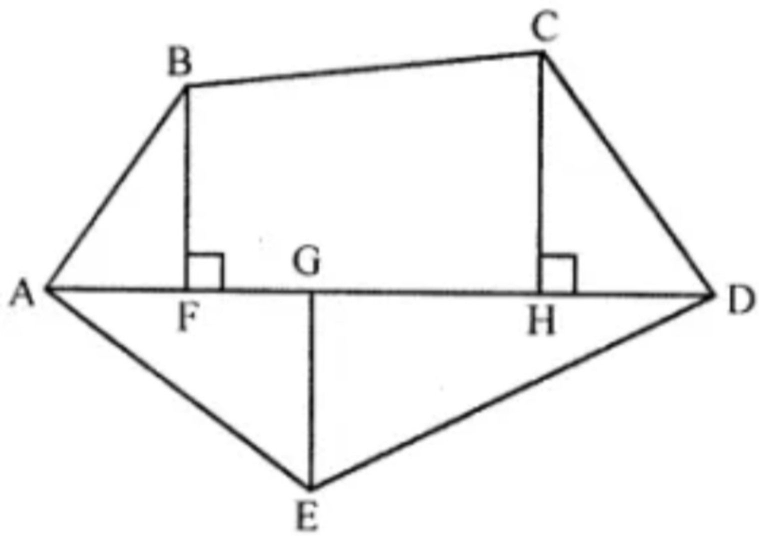
$\therefore$ Area of rt. Δ BEC $= \frac{1}{2} \times 6 \times 8 = 24 cm^2$

Area of pentagon ABECD = Area of rectangle – area of Δ

$= (180 - 24)\ cm^2 = 156\ cm^2$

Question 16.

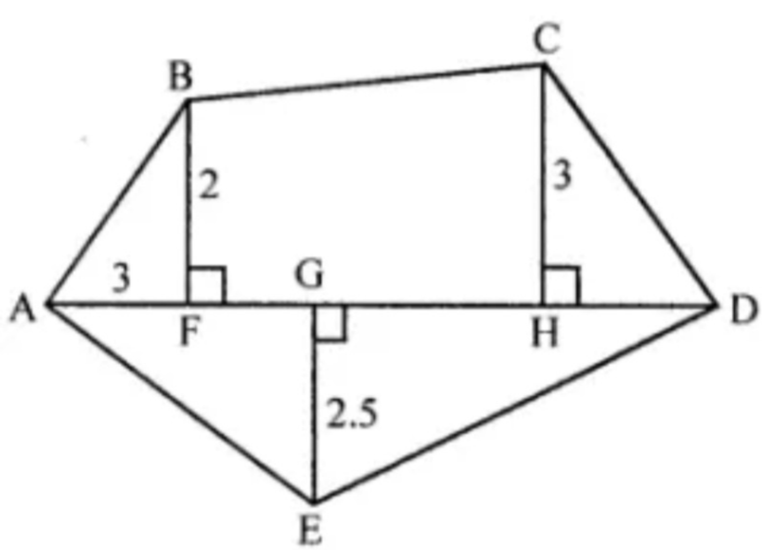
Polygon ABCDE is divided into parts as shown in the given figure. Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.



Solution:

In the given figure, ABCDE, AD = 8 cm, AH = 6 cm, AG = 4 cm,

AF = 3cm ⊥ BF = 2 cm CH = 3 cm and ⊥ EG = 2.5 cm



The given figure, consists of 3 triangles and one trapezium.

Now area of $\Delta AED = \frac{1}{2} AD \times GE$

$= \frac{1}{2} \times 8 \times 2.5 = 10 \text{ cm}^2$

Area of $\Delta ABF = \frac{1}{2} AF \times BF$

$= \frac{1}{2} \times 3 \times 2 = 3 \text{ cm}^2$

Area of $\Delta CDH = \frac{1}{2} \times HD \times CH$

$= \frac{1}{2}(AD - AH) \times 3$

$= \frac{1}{2}(8 - 6) \times 3$

$= \frac{1}{2} \times 2 \times 3 = 3 \text{ cm}^2$

Area of trapezium BFHC

$= \frac{1}{2}(BF + CH) \times FH$

$= \frac{1}{2} (2 + 3) \times (AH - AF)$

$= \frac{1}{2} \times 5 \times (6 - 3)$

$= \frac{1}{2} \times 5 \times 3 = 7.5 \text{ cm}^2$

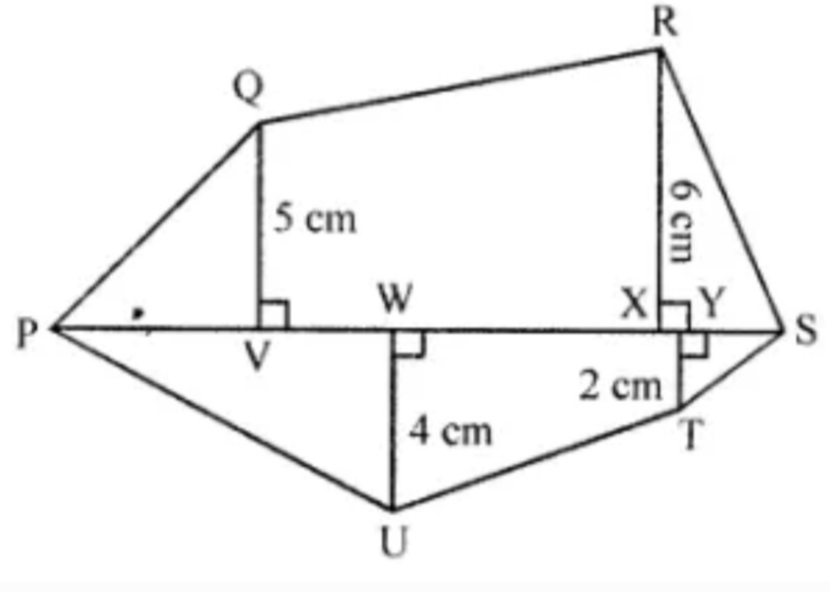
$\therefore$ Total area of the figure = Area of ΔAED + Area of ΔABF

+ Area of ΔCDH + Area of trapezium BFHC

$= 10 + 3 + 3 + 7.5 = 23.5 \text{ cm}^2$

Question 17.

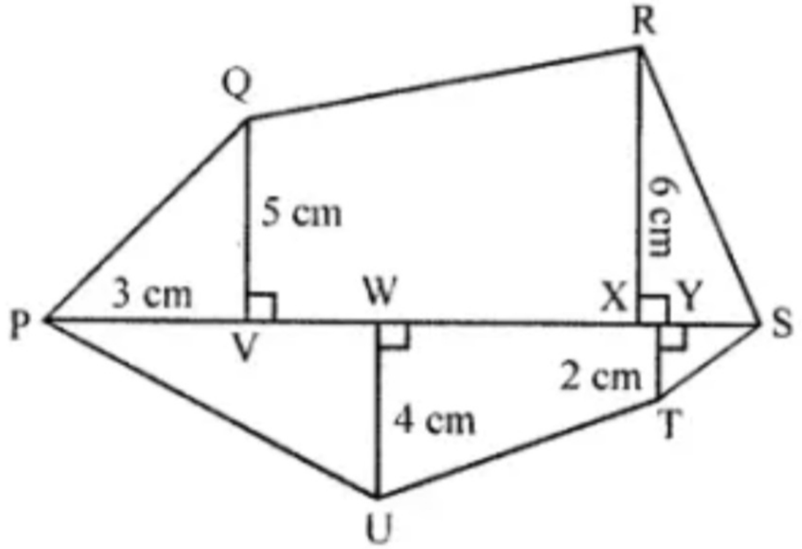
Find the area of polygon PQRSTU shown in 1 the given figure, if PS = 11 cm, PY = 9 cm, PX = 8 cm, PW = 5 cm, PV = 3 cm, QV = 5 cm, UW = 4 cm, RX = 6 cm, TY = 2 cm.



Solution:

In the figure, PQRSTU, in which PS = 11 cm, PY = 9 cm, PX = 8 cm,

PW = 5 cm, PV = 3 cm, QV = 5 cm, UW = 4 cm, RX = 6 cm, TY = 2 cm



The figure, consists of 4 triangle and 2 trapeziums

VX = PX − PV

= 8 − 3 = 5 cm

XS = PS − PX

= 11 − 8 = 3 cm

YS = PS − PY

= 11 − 9 = 2 cm

WY = PY − PW

= 9 − 5 = 4 cm

Now area $\Delta PQV = \frac{1}{2} PV + QV$

$= \frac{1}{2} \times 3 \times 5 = \frac{15}{2} = 7.5 \text{ cm}^2$

Area of $\Delta RXS = \frac{1}{2} \times S \times R$

$= 3 \times 6 = 9 \text{ cm}^2$

Area $\Delta PUW = \frac{1}{2} \times PW \times UW$

$= \frac{1}{2} \times 5 \times 4 = 10 \text{ cm}^2$

Area $\Delta YTS = \frac{1}{2} \times YS \times TY$

$= \frac{1}{2} \times 2 \times 2 = 2 \text{ cm}^2$

Area of trapezium ΔVX R

$= \frac{1}{2}(QV + RX) \times VX$

$= \frac{1}{2} (5 + 6) \times 5 = \frac{1}{2} \times 11 \times 5 \text{ cm}^2$

$= \frac{55}{7} = 27.5 \text{ cm}^2$

Area of trapezium WUTY

$= \frac{1}{2}(UW + TY) \times WY$

$= \frac{1}{2}(4 + 2) \times 4 = \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$

Now area of the figure = 7.5 + 9 + 10 + 2 + 27.5 + 12 cm^2 = 68 cm^2