EXERCISE 18.1

1. Find the values of
(i) 7 sin 30^0 cos 60^0
(ii) 3 $\sin^2 45^0$ + 2 $\cos^2 60^0$
(iii) $\cos^2 45^0$ + $\sin^2 60^0$ + $\sin^2 30^0$
(iv) cos 90^0 + $\cos^2 45^0$ sin 30^0 tan 45^0.
Solution:

(i) 7 sin 30^0 cos 60^0
Substituting the values
= 7 × ½ × ½
= (7 × 1 × 1)/(2 × 2)
= 7/4

(ii) 3 $\sin^2 45^0$ + 2 $\cos^2 60^0$
Substituting the values
= 3 × $(1/\sqrt{2})^2$ + 2 × $(1/2)^2$
By further calculation
= 3 × ½ + 2 × ¼
= 3/2 + ½
So we get
= (3 + 1)/2
= 4/2
= 2

(iii) $\cos^2 45^0$ + $\sin^2 60^0$ + $\sin^2 30^0$
Substituting the values
= $(1/\sqrt{2})^2 + (\sqrt{3}/2)^2 + (1/2)^2$
By further calculation
= ½ + ¾ + ¼
Taking LCM
= (2 + 3 + 1)/4
= 6/4
= 3/2

(iv) cos 90^0 + $\cos^2 45^0$ sin 30^0 tan 45^0
Substituting the values
= 0 + $(1/\sqrt{2})^2$ × ½ × 1
By further calculation
= ½ × ½ × 1
= ¼

2. Find the values of

$$(i)\frac{sin^2 45^0 + cos^2 45^0}{tan^2 60^0}$$

$$(ii)\frac{sin30^0 - sin90^0 + 2cos0^0}{tan30^0 \times tan60^0}$$

$$(iii)\frac{4}{3}tan^2 30^0 + sin^2 60^0 - 3cos^2 60^0 + \frac{3}{4}tan^2 60^0 - 2tan^2 45^0.$$

Solution:

$$(i)\frac{sin^2 45^0 + cos^2 45^0}{tan^2 60^0}$$

$Substituting\ the\ values$

$$= \frac{(\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2}{\sqrt{3}^2}$$

$By\ further\ calculation$

$$= \frac{\frac{1}{2} + \frac{1}{2}}{3}$$

$$= \frac{1}{3}$$

$$(ii)\frac{sin30^0 - sin90^0 + 2cos0^0}{tan30^0 \times tan60^0}$$

$Substituting\ the\ values$

$$= \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{\frac{1}{2} - 1 + 2}{1}$$

$So\ we\ get$

$$= \frac{1}{2} - 1 + 2$$

$$= \frac{1}{2} + 1$$

$$= \frac{1+2}{2}$$

$$= \frac{3}{2}$$

(iii) $4/3 \tan^2 30^0 + \sin^2 60^0 - 3 \cos^2 60^0 + ¾ \tan^2 60^0 - 2 \tan^2 45^0$
Substituting the values
$= 4/3 (1/\sqrt{3})^2 + (\sqrt{3}/2)^2 - 3 (1/2)^2 + ¾ \times (\sqrt{3})^2 - 2 \times 1^2$
By further calculation
= 4/3 × 1/3 + ¾ - 3 × ¼ + ¾ × 3 – 2 × 1
= 4/9 + ¾ - 3/4 + 9/4 – 2
So we get
= 4/9 + 9/4 – 2
Taking LCM
= (16 + 81 – 72)/36
= (97 – 72)/36
= 25/36

3. Find the values of

$$(i)\frac{sin60^0}{cos^2 45^0} - 3tan30^0 + 5cos90^0$$

$$(ii)2\sqrt{2}cos45^0 cos60^0 + 2\sqrt{3}sin30^0 tan60^0 - cos0^0$$

$$(iii)\frac{4}{5}tan^2 60^0 - \frac{2}{sin^2 30^0} - \frac{3}{4}tan^2 30^0.$$

Solution:

$$(i)\frac{sin60^0}{cos^2 45^0} - 3tan30^0 + 5cos90^0$$

$$Substituting\ the\ values$$

$$= \frac{(\frac{\sqrt{3}}{2})}{(\frac{1}{\sqrt{2}})^2} - 3 \times \frac{1}{\sqrt{3}} + 5 \times 0$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} - \sqrt{3} - 0$$

$$So\ we\ get$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{1} - \sqrt{3}$$

$$= \sqrt{3} - \sqrt{3}$$

$$= 0$$

(ii) $2\sqrt{2} \cos 45^0 \cos 60^0 + 2\sqrt{3} \sin 30^0 \tan 60^0 - \cos 0^0$
Substituting the values

= 2√2 × 1/√2 × ½ + 2√3 × ½ × √3 – 1
By further calculation
= 2 × 1/1 × 1/2 + 2 × 3 × ½ - 1
= 1 + 3 – 1
= 3

$$(iii) \frac{4}{5} tan^2 60^0 + \frac{2}{sin^2 30^0} - \frac{3}{4} tan^2 30^0$$

Substituting the values

$$= \frac{4}{5} \times (\sqrt{3})^2 - \frac{2}{(\frac{1}{2})^2} - \frac{3}{4}(\frac{1}{\sqrt{3}})^2$$

By further calculation

$$= \frac{4}{5} \times 3 - \frac{2}{\frac{1}{4}} - \frac{3}{4} \times \frac{1}{3}$$

So we get

$$= \frac{12}{5} - \frac{2 \times 4}{1} - \frac{1}{4}$$

$$= \frac{12}{5} - \frac{8}{1} - \frac{1}{4}$$

Taking LCM

$$= \frac{48 - 160 - 5}{20}$$

$$= \frac{43 - 160}{20}$$

$$= \frac{-117}{20}$$

$$= -5\frac{17}{20}$$

4. Prove that
(i) $\cos^2 30^0 + \sin 30^0 + \tan^2 45^0 = 2 \frac{1}{4}$
(ii) $4 (\sin^4 30^0 + \cos^4 60^0) – 3 (\cos^2 45^0 – \sin^2 90^0) = 2$
(iii) $\cos 60^0 = \cos^2 30^0 – \sin^2 30^0$.
Solution:

(i) $\cos^2 30^0 + \sin 30^0 + \tan^2 45^0 = 2 \frac{1}{4}$

Consider
LHS = $\cos^2 30^0 + \sin 30^0 + \tan^2 45^0$
Substituting the values
= $(\sqrt{3}/2)^2 + ½ + 1^2$
By further calculation
= ¾ + ½ + 1
Taking LCM
= (3 + 2 + 4)/4
= 9/4
= 2 ¼
= RHS

Therefore, LHS = RHS.

(ii) $4 (\sin^4 30^0 + \cos^4 60^0) – 3 (\cos^2 45^0 – \sin^2 90^0) = 2$
Consider
LHS = $4 (\sin^4 30^0 + \cos^4 60^0) – 3 (\cos^2 45^0 – \sin^2 90^0)$
Substituting the values
= $4[(½)^4 + (½)^4] – 3 [(1/\sqrt{2})^2 – 1^2]$
It can be written as
= 4[½ × ½ × ½ × ½ + ½ × ½ × ½ × ½] - 3 [½ - 1]
By further calculation
= 4 [1/16 + 1/16] – 3 (- ½)
= 4[(1 + 1)/16] + 3/2
So we get
= (4 × 3)/16 + 3/2
= 8/16 + 3/2
= ½ + 3/2
= (1 + 3)/2
= 4/2
= 2
= RHS

Therefore, LHS = RHS.

(iii) $\cos 60^0 = \cos^2 30^0 – \sin^2 30^0$
Consider
LHS = $\cos 60^0$ = ½
RHS = $\cos^2 30^0 – \sin^2 30^0$
Substituting the values
= $(\sqrt{3}/2)^2 + (1/2)^2$
By further calculation
= ¾ - ¼
= (3 – 1)/4
= 2/4
= ½

= RHS

Therefore, LHS = RHS.

5. (i) If x = 30^0, verify that tan 2x = 2tanx/ (1- $\tan^2$ x).
(ii) If x = 15^0, verify that 4 sin 2x cos 4x sin 6x = 1.
Solution:

(i) It is given that
x = 30^0
Consider LHS = tan 2x
Substituting the value of x
= tan 60^0
= √3

$$RHS = \frac{2tanx}{1 - tan^2x}$$

$$Substituting\ the\ value\ of\ x$$

$$= \frac{2tan30^0}{1 - tan^2 30^0}$$

$$By\ further\ calculation$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - (\frac{1}{\sqrt{3}})^2}$$

$$So\ we\ get$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

Therefore, LHS = RHS.

(ii) It is given that
$x = 15^0$
$2x = 15 \times 2 = 30^0$
$4x = 15 \times 4 = 60^0$
$6x = 15 \times 6 = 90^0$

Here
LHS = 4 sin 2x cos 4x sin 6x
It can be written as
$= 4 \sin 30^0 \cos 60^0 \sin 90^0$
So we get
$= 4 \times ½ \times ½ \times 1$
$= 1$
= RHS

Therefore, LHS = RHS.

6. Find the values of

$$(i) \sqrt{\frac{1 - cos^2 30^0}{1 - sin^2 30^0}}$$

$$(ii) \frac{sin45^0 cos45^0 cos60^0}{sin60^0 cos30^0 tan45^0}.$$

Solution:

$$(i) \sqrt{\frac{1 - cos^2 30^0}{1 - sin^2 30^0}}$$

Substituting the values

$$= \sqrt{\frac{1 - (\frac{\sqrt{3}}{2})^2}{1 - (\frac{1}{2})^2}}$$

By further calculation

$$= \sqrt{\frac{1 - \frac{3}{4}}{1 - \frac{1}{4}}}$$

Taking LCM

$$= \sqrt{\frac{\frac{4-3}{4}}{\frac{4-1}{4}}}$$

$$= \sqrt{\frac{\frac{1}{4}}{\frac{3}{4}}}$$

$$= \sqrt{\frac{1}{4} \times \frac{4}{3}}$$

So we get

$$= \sqrt{\frac{1}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$(ii) \frac{sin45^0 cos45^0 cos60^0}{sin60^0 cos30^0 tan45^0}$$

Substituting the values

$$= \frac{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{1}{2}}{\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times 1}$$

By further calculation

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{3}{4} \times 1}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

7. If θ = 30^0, verify that
(i) sin 2θ = 2 sin θ cos θ
(ii) cos 2θ = 2 $\cos^2$ θ – 1
(iii) sin 3θ = 3 sin θ – 4 $\sin^3$ θ
(iv) cos 3θ = 4 $\cos^3$ θ – 3 cos θ.
Solution:

It is given that θ = 30^0
(i) sin 2θ = 2 sin θ cos θ
Consider
LHS = sin 2θ

Substituting the value of θ
$= \sin 2 \times 30^0$
$= \sin 60^0$
$= \sqrt{3}/2$

RHS = $2 \sin \theta \cos \theta$
Substituting the value of θ
$= 2 \sin 30^0 \cos 30^0$
So we get
$= 2 \times ½ \times \sqrt{3}/2$
$= 1 \times \sqrt{3}/2$
$= \sqrt{3}/2$

Therefore, LHS = RHS.

(ii) $\cos 2\theta = 2 \cos^2 \theta – 1$
Consider
LHS = $\cos 2\theta$
Substituting the value of θ
$= \cos 2 \times 30^0$
$= \cos 60^0$
= ½

RHS = $2 \cos^2 \theta – 1$
Substituting the value of θ
$= 2 \cos^2 30^0 – 1$
So we get
$= 2 (\sqrt{3}/2)^2 – 1$
$= 2 \times ¾ - 1$
$= 3/2 – 1$
$= (3 – 2)/2$
= ½

Therefore, LHS = RHS.

(iii) $\sin 3\theta = 3 \sin \theta – 4 \sin^3 \theta$
Consider
LHS = $\sin 3\theta$
Substituting the value of θ
$= \sin 3 \times 30^0$
$= \sin 90^0$
= 1

RHS = $3 \sin \theta – 4 \sin^3 \theta$
Substituting the value of θ
$= 3 \sin 30^0 – 4 \sin^3 30^0$

So we get
$= 3 \times ½ - 4 \times (1/2)^3$
$= 3/2 – 4 \times 1/8$
$= 3/2 – ½$
Taking LCM
$= (3 – 1)/2$
$= 2/2$
$= 1$

Therefore, LHS = RHS.

(iv) $\cos 3\theta = 4 \cos^3 \theta – 3 \cos \theta$
Consider
LHS = $\cos 3\theta$
Substituting the value of θ
$= \cos 3 \times 30^0$
$= \cos 90^0$
$= 0$

RHS = $4 \cos^3 \theta – 3 \cos \theta$
Substituting the value of θ
$= 4 \cos^3 30^0 – 3 \cos 30^0$
So we get
$= 4 \times (\sqrt{3}/2)^3 – 3 \times (\sqrt{3}/2)$
By further calculation
$= 4 \times 3\sqrt{3}/8 - 3\sqrt{3}/2$
$= 3\sqrt{3}/2 - 3\sqrt{3}/2$
$= 0$

Therefore, LHS = RHS.

8. If $\theta = 30^0$, find the ratio 2 sin θ: sin 2 θ.
Solution:

It is given that $\theta = 30^0$
We know that
$2 \sin \theta: \sin 2\theta = 2 \sin 30^0: \sin 2 \times 30^0$
So we get
$= 2 \sin 30^0: \sin 60^0$
$= 2 \sin 30^0/ \sin 60^0$
Substituting the values

$$= \frac{2 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$By\ further\ simplification$

$$= \frac{1}{\frac{\sqrt{3}}{2}}$$
$$= \frac{1 \times 2}{\sqrt{3}}$$
$$= \frac{2}{\sqrt{3}}$$
= 2: √3

Therefore, 2 sin θ: sin 2θ = 2: √3.

9. By means of an example, show that sin (A + B) ≠ sin A + sin B.
Solution:

Consider A = 30^0 and B = 60^0
LHS = sin (A + B)
Substituting the values of A and B
= sin (30^0 + 60^0)
= sin 90^0
= 1

RHS = sin A + sin B
Substituting the values
= sin 30^0 + sin 60^0
So we get
= ½ + √3/2
= (1 + √3)/2

Therefore, LHS ≠ RHS i.e. sin (A + B) ≠ sin A + sin B.

10. If A = 60^0 and B = 30^0, verify that
(i) sin (A + B) = sin A cos B + cos A sin B
(ii) cos (A + B) = cos A cos B – sin A sin B
(iii) sin (A – B) = sin A cos B – cos A sin B
(iv) tan (A – B) = (tan A – tan B)/ (1 + tan A tan B).
Solution:

It is given that A = 60^0 and B = 30^0
(i) sin (A + B) = sin A cos B + cos A sin B
Here

LHS = sin (A + B)
Substituting the values of A and B
= sin (60^0 + 30^0)
= sin 90^0
= 1

RHS = sin A cos B + cos A sin B
Substituting the values of A and B
= sin 60^0 cos 30^0 + cos 60^0 sin 30^0
So we get
= √3/2 × √3/2 + ½ × ½
By further calculation
= ¾ + ¼
= 4/4
= 1

Therefore, LHS = RHS.

(ii) cos (A + B) = cos A cos B – sin A sin B
Here
LHS = cos (A + B)
Substituting the value of A and B
= cos (60^0 + 30^0)
= cos 90^0
= 0

RHS = cos A cos B – sin A sin B
Substituting the value of A and B
= cos 60^0 cos 30^0 – sin 60^0 sin 30^0
So we get
= ½ × √3/2 - √3/2 + ½
= √3/4 - √3/4
= 0

Therefore, LHS = RHS.

(iii) sin (A – B) = sin A cos B – cos A sin B
Here
LHS = sin (A – B)
Substituting the values of A and B
= sin (60^0 – 30^0)
= sin 30^0
= ½

RHS = sin A cos B – cos A sin B
Substituting the values of A and B

$= \sin 60^0 \cos 30^0 – \cos 60^0 \sin 30^0$
So we get
$= \sqrt{3}/2 \times \sqrt{3}/2 – ½ \times ½$
$= ¾ - ¼$
$= (3 – 1)/ 4$
$= 2/4$
$= ½$

Therefore, LHS = RHS.

(iv) tan (A – B) = (tan A – tan B)/ (1 + tan A tan B)
Here
LHS = tan (A – B)
Substituting the values of A and B
$= \tan (60^0 – 30^0)$
$= \tan 30^0$
$= 1/\sqrt{3}$

RHS = (tan A – tan B)/ (1 + tan A tan B)
Substituting the values of A and B
$= (\tan 60^0 – \tan 30^0)/ (1 + \tan 60^0 \tan 30^0)$
So we get

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$By\ further\ simplification$$

$$= \frac{3-1}{\frac{\sqrt{3}}{1+1}}$$

$$= \frac{2}{\frac{\sqrt{3}}{2}}$$

We get
$= 2/\sqrt{3} \times ½$
$= 1/\sqrt{3}$

Therefore, LHS = RHS.

11. (i) If 2θ is an acute angle and 2 sin 2θ = √3, find the value of θ.
(ii) If $20^0 + x$ is an acute angle and $\cos (20^0 + x) = \sin 60^0$, then find the value of x.
(iii) If $3 \sin^2 \theta = 2 ¼$ and θ is less than 90^0, find the value of θ.
Solution:

(i) It is given that
2θ is an acute angle

$2 \sin 2\theta = \sqrt{3}$
It can be written as
$\sin 2\theta = \sqrt{3}/2 = \sin 60^0$
By comparing
$2\theta = 60^0$
So we get
$\theta = 60^0/2 = 30^0$

Therefore, $\theta = 30^0$.

(ii) It is given that
$20^0 + x$ is an acute angle
$\cos (20^0 + x) = \sin 60^0$
It can be written as
$\cos (20^0 + x) = \sin 60^0 = \cos (90^0 – 60^0)$
$= \cos 30^0$
By comparing
$20^0 + x = 30^0$
$x = 30^0 – 20^0 = 10^0$

Therefore, $x = 10^0$.

(iii) It is given that
$3 \sin^2 \theta = 2 \frac{1}{4}$
θ is less than 90^0
We can write it as
$\sin^2 \theta = 9/(4 \times 3) = \frac{3}{4}$
So we get
$\sin \theta = \sqrt{3}/2 = \sin 60^0$
By comparing
$\theta = 60^0$

Therefore, $\theta = 60^0$.

12. If θ is an acute angle and sin θ = cos θ, find the value of θ and hence, find the value of 2 $\tan^2 \theta$ + $\sin^2 \theta$ – 1.
Solution:

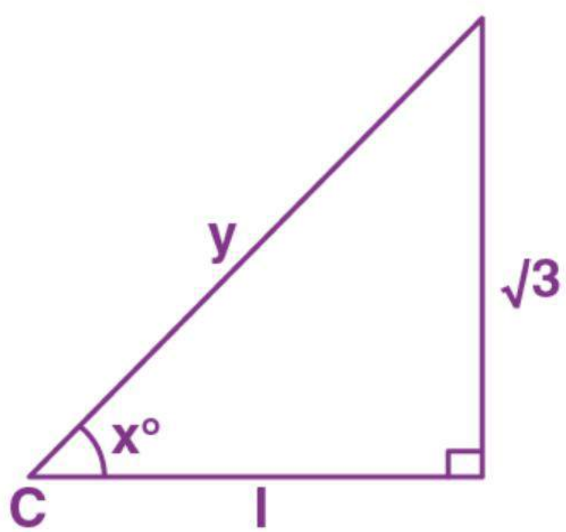
It is given that
$\sin \theta = \cos \theta$
We can write it as
$\sin \theta / \cos \theta = 1$
$\tan \theta = 1$
We know that $\tan 45^0 = 1$
$\tan \theta = \tan 45^0$
So we get

$\theta = 45^0$

We know that
$2\tan^2\theta + \sin^2\theta - 1 = 2\tan^2 45^0 + \sin^2 45^0 - 1$
Substituting the values
$= 2(1)^2 + (1/\sqrt{2})^2 - 1$
By further calculation
$= 2 \times 1 \times 1 + ½ - 1$
$= 2 + ½ - 1$
$= 5/2 - 1$
Taking LCM
$= (5 - 2)/2$
$= 3/2$

Therefore, $2\tan^2\theta + \sin^2\theta - 1 = 3/2$.

13. From the adjoining figure, find
(i) tan x^0
(ii) x
(iii) cos x^0
(iv) use sin x^0 to find y.



Solution:

(i) $\tan x^0$ = perpendicular/base
It can be written as
= AB/BC
$= \sqrt{3}/1$
$= \sqrt{3}$

(ii) $\tan x^0 = \sqrt{3}$
We know that $\tan 60^0 = \sqrt{3}$

$\tan x^0 = \tan 60^0$
$x = 60$

(iii) We know that
$\cos x^0 = \cos 60^0$
So we get
$\cos x^0 = ½$

(iv) $\sin x^0$ = perpendicular/hypotenuse = AB/AC
Substitute x = 60 from (ii)
$\sin 60^0 = \sqrt{3}/y$
We know that $\sin 60^0 = \sqrt{3}/2$
$\sqrt{3}/2 = \sqrt{3}/y$
By further calculation
$y = (\sqrt{3} \times 2)/ \sqrt{3}$
$y = (2 \times 1)/1 = 2$

Therefore, y = 2.

14. If 3θ is an acute angle, solve the following equations for θ:
(i) $2 \sin 3\theta = \sqrt{3}$
(ii) $\tan 3\theta = 1$.
Solution:

(i) $2 \sin 3\theta = \sqrt{3}$
It can be written as
$\sin 3\theta = \sqrt{3}/2$
We know that $\sin 60^0 = \sqrt{3}/2$
$\sin 3\theta = \sin 60^0$
$3\theta = 60^0$
So we get
$\theta = 60/3 = 20^0$

(ii) $\tan 3\theta = 1$
We know that $\tan 45^0 = 1$
$\tan 3\theta = \tan 45^0$
So we get
$3\theta = 45^0$
$\theta = 15^0$

15. If $\tan 3x = \sin 45^0 \cos 45^0 + \sin 30^0$, find the value of x.
Solution:

We know that
$\tan 3x = \sin 45^0 \cos 45^0 + \sin 30^0$
Substituting the values

$= 1/\sqrt{2} \times 1/\sqrt{2} + ½$
$= ½ + ½$
$= 1$

We know that
$\tan 3x = \tan 45^0$
By comparing
$3x = 45^0$
$x = 45/3 = 15^0$

Therefore, the value of x is 15^0.

16. If $4 \cos^2 x^0 – 1 = 0$ and $0 \le x \le 90$, find
(i) x
(ii) $\sin^2 x^0 + \cos^2 x^0$
(iii) $\cos^2 x^0 – \sin^2 x^0$.
Solution:

It is given that
$4 \cos^2 x^0 – 1 = 0$
$4\cos^2 x^0 = 1$
It can be written as
$\cos^2 x^0 = ¼$
$\cos x^0 = \pm \sqrt{1/4}$
$\cos x^0 = + \sqrt{1/4}$ [$0 \le x \le 90^0$, then $\cos x^0$ is positive]
$\cos x^0 = ½$
We know that $\cos 60^0 = ½$
$\cos x^0 = \cos 60^0$
By comparing
$x = 60$

(ii) $\sin^2 x^0 + \cos^2 x^0 = \sin^2 60^0 + \cos^2 60^0$
Substituting the values
$= (\sqrt{3}/2)^2 + (1/2)^2$
By further calculation
$= ¾ + ¼$
$= (3 + 1)/4$
$= 4/4$
$= 1$

Therefore, $\sin^2 x^0 + \cos^2 x^0 = 1$.

(iii) $\cos^2 x^0 – \sin^2 x^0 = \cos^2 60^0 – \sin^2 60^0$
Substituting the values
$= (1/2)^2 – (\sqrt{3}/2)^2$
By further calculation

$= \frac{1}{4} - \sqrt{3}/2 \times \sqrt{3}/2$
$= \frac{1}{4} - \frac{3}{4}$
$= (1 - 3)/4$
$= -2/4$
$= -\frac{1}{2}$

Therefore, $\cos^2 x^0 - \sin^2 x^0 = -\frac{1}{2}$.

17. (i) If sec θ = cosec θ and $0^0 \le \theta \le 90^0$, find the value of θ.
(ii) If tan θ = cot θ and $0^0 \le \theta \le 90^0$, find the value of θ
Solution:

(i) It is given that
$\sec \theta = \text{cosec } \theta$
We know that
$\sec \theta = 1/\cos \theta$
$\text{cosec } \theta = 1/\sin \theta$
So we get
$1/\cos \theta = 1/\sin \theta$
$\sin \theta/\cos \theta = 1$
$\tan \theta = 1$
Here $\tan 45^0 = 1$
$\tan \theta = \tan 45^0$
$\theta = 45^0$

(ii) It is given that
$\tan \theta = \cot \theta$
We know that $\cot \theta = 1/\tan \theta$
$\tan \theta = 1/\tan \theta$
So we get
$\tan^2 \theta = 1$
$\tan \theta = \pm \sqrt{1}$
$\tan \theta = +1$ [$0 \le \theta \le 90^0$, tan θ is positive]
$\tan \theta = \tan 45^0$
By comparing
$\theta = 45^0$

18. If sin 3x = 1 and $0^0 \le 3x \le 90^0$, find the values of
(i) sin x
(ii) cos 2x
(iii) $\tan^2 x - \sec^2 x$.
Solution:

It is given that
$\sin 3x = 1$
We know that $\sin 90^0 = 1$
$\sin 3x = \sin 90^0$

By comparing
$3x = 90$
$x = 90/3$
$x = 30^0$

(i) $\sin x = \sin 30^0 = 1/2$

(ii) $\cos 2x = \cos 2 \times 30 = \cos 60^0 = 1/2$

(iii) $\tan^2 x - \sec^2 x = \tan^2 30^0 - \sec^2 30^0$
Substituting the values
$= (1/\sqrt{3})^2 - (2/\sqrt{3})^2$
By further calculation
$= 1/3 - 4/3$
$= (1 - 4)/3$
$= - 3/3$
$= -1$

Therefore, $\tan^2 x - \sec^2 x = - 1$.

19. If $3 \tan^2 \theta - 1 = 0$, find $\cos 2\theta$, given that θ is acute.
Solution:

It is given that
$3 \tan^2 \theta - 1 = 0$
We can write it as
$3 \tan^2 \theta = 1$
$\tan^2 \theta = 1/3$
$\tan \theta = 1/\sqrt{3}$ [θ is acute so $\tan \theta$ is positive]
$\theta = 30^0$

So we get
$\cos 2\theta = \cos 2 \times 30^0 = \cos 60^0 = ½$

20. If $\sin x + \cos y = 1$, $x = 30^0$ and y is acute angle, find the value of y.
Solution:

It is given that
$\sin x + \cos y = 1$
$x = 30^0$
Substituting the values
$\sin 30^0 + \cos y = 1$
$1/2 + \cos y = 1$
It can be written as
$\cos y = 1 - ½$
Taking LCM
$\cos y = (2 - 1)/2 = ½$

We know that $\cos 60^0 = ½$
$\cos y = \cos 60^0$
So we get
$y = 60^0$

21. If $\sin (A + B) = \sqrt{3}/2 = \cos (A – B)$, $0^0 < A + B \leq 90^0$ $(A > B)$, find the values of A and B.
Solution:

It is given that
$\sin (A + B) = \sqrt{3}/2 = \cos (A – B)$
Consider
$\sin (A + B) = \sqrt{3}/2$
We know that $\sin 60^0 = \sqrt{3}/2$
$\sin (A + B) = \sin 60^0$
$A + B = 60^0$ …… (1)
Similarly
$\cos (A – B) = \sqrt{3}/2$
We know that $\cos 30^0 = \sqrt{3}/2$
$\cos (A – B) = \cos 30^0$
$A – B = 30^0$ ……. (2)

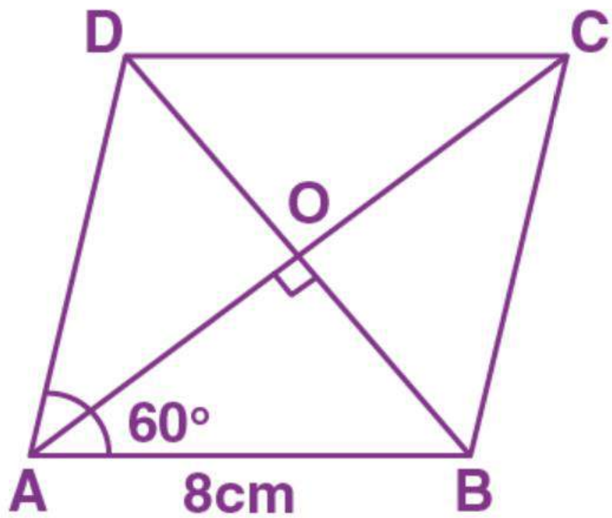
By adding both the equations
$A + B + A – B = 60^0 + 30^0$
So we get
$2A = 90^0$
$A = 90^0/2 = 45^0$

Now substitute the value of A in equation (1)
$45^0 + B = 60^0$
By further calculation
$B = 60^0 – 45^0 = 15^0$

Therefore, $A = 45^0$ and $B = 15^0$.

22. If the length of each side of a rhombus is 8 cm and its one angle is 60^0, then find the lengths of the diagonals of the rhombus.
Solution:

It is given that
Each side of a rhombus = 8 cm
One angle = 60^0



We know that the diagonals bisect the opposite angles
$\angle OAB = 60^0/2 = 30^0$

In right $\angle AOB$
$\sin 30^0 = OB/AB$
So we get
$½ = OB/8$
By further calculation
$OB = 8/2 = 4$ cm
$BD = 2OB = 2 \times 4 = 8$ cm

$\cos 30^0 = AO/AB$
Substituting the values
$\sqrt{3}/2 = AO/8$
By further calculation
$AO = 8\sqrt{3}/2 = 4\sqrt{3}$

Here
$AC = 4\sqrt{3} \times 2 = 8\sqrt{3}$ cm
Therefore, the length of the diagonals of the rhombus are 8 cm and $8\sqrt{3}$ cm.

23. In the right-angled triangle ABC, $\angle C = 90^0$ and $\angle B = 60^0$. If AC = 6 cm, find the lengths of the sides BC and AB.
Solution:

In the right-angled triangle ABC, $\angle C = 90^0$ and $\angle B = 60^0$
AC = 6 cm

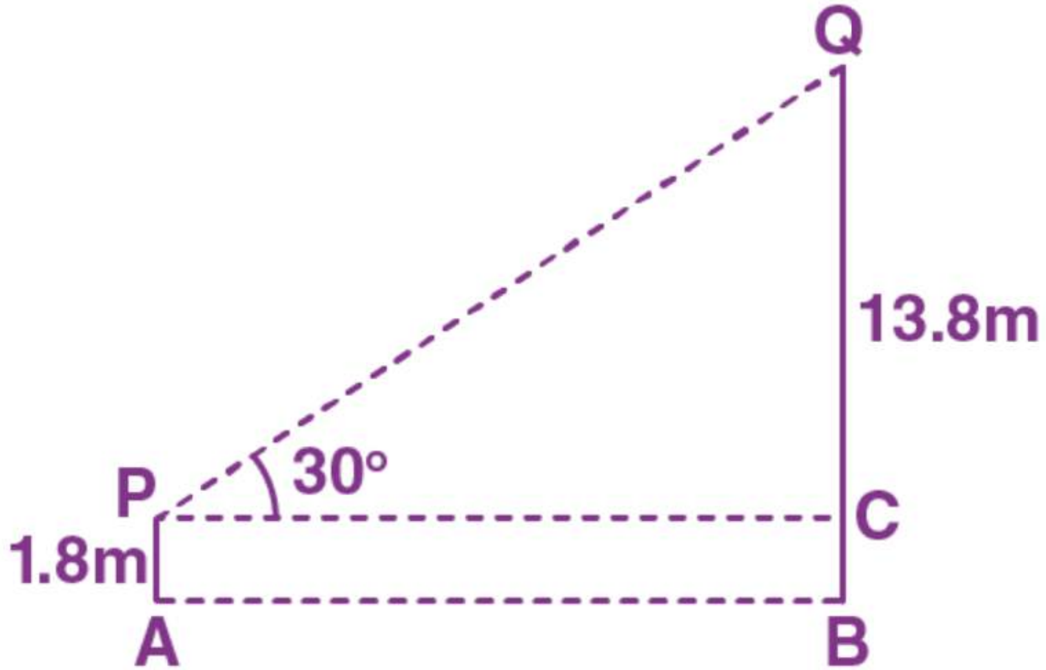
We know that

tan B = AC/BC
Substituting the values
tan 60^0 = 6/BC
So we get
√3 = 6/BC
BC = 6/√3
It can be written as
= 6√3/(√3 + √3)
= 6√3/3
= 2√3 cm

sin 60^0 = AC/AB
Substituting the values
√3/2 = 6/AB
By further calculation
AB = (6 × 2)/ √3
So we get
AB = (12 × √3)/(√3 × √3)
= 12√3/3
= 4√3 cm

Therefore, the lengths of the sides BC = 2√3 cm and AB = 4√3 cm.

24. In the adjoining figure, AP is a man of height 1.8 m and BQ is a building 13.8 m high. If the man sees the top of the building by focusing his binoculars at an angle of 30^0 to the horizontal, find the distance of the man from the building.



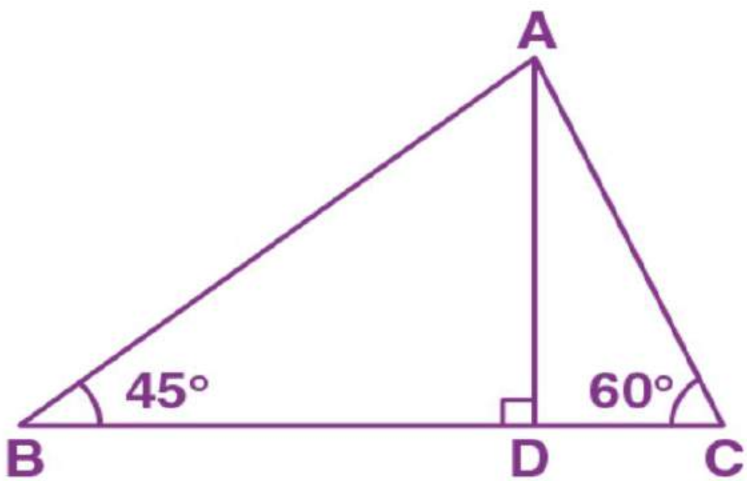
Solution:

It is given that
Height of man AP = 1.8 m
Height of building BQ = 13.8 m
Angle of elevation from the top of the building to the man = 30^0
Consider AB as the distance of the man from the building

AB = x then PC = x
QC = 13.8 – 1.8 = 12 m
In right △ PQC
tan θ = QC/PC
Substituting the values
tan 30^0 = 12/x
By further calculation
1/√3 = 12/x
x = 12 √3 m

Therefore, the distance of the man from the building is 12 √3 m.

25. In the adjoining figure, ABC is a triangle in which ∠B = 45^0 and ∠C = 60^0. If AD ⊥ BC and BC = 8 m, find the length of the altitude AD.



Solution:

In triangle ABC
∠B = 45^0 and ∠C = 60^0
AD ⊥ BC and BC = 8 m

In right △ ABD
tan 45^0 = AD/BD
So we get
1 = AD/BD
AD = BD

In right △ ACD
tan 60^0 = AD/DC
So we get
√3 = AD/DC
DC = AD/√3

Here
BD + DC = AD + AD/√3
Taking LCM
BC = (√3AD + AD)/ √3
8 = [AD (√3 + 1)]/ √3
By further calculation
AD = 8√3/(√3 + 1)
It can be written as

$$= \frac{8\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

So we get

$$= \frac{8(3-\sqrt{3})}{3-1}$$

$$= \frac{8(3-\sqrt{3})}{2}$$

= 4 (3 - √3) m

Therefore, the length of the altitude AD is 4 (3 - √3) m.

EXERCISE 18.2

Without using trigonometric tables, evaluate the following (1 to 5):
1. (i) cos 18^0/sin 72^0
(ii) tan 41^0/cot 49^0
(iii) cosec $17^0$30'/sec 72^0 30'
Solution:

(i) cos 18^0/sin 72^0
It can be written as
= cos 18^0/ sin (90^0 – 18^0)
So we get
= cos 18^0/cos 18^0
= 1

(ii) tan 41^0/cot49^0
It can be written as
= tan 41^0/ cot (90^0 – 41^0)
So we get
= tan 41^0/ tan 41^0
= 1

(iii) cosec $17^0$30'/sec 72^0 30'
It can be written as
= cosec 17^0 30'/ sec (90^0 – 17^0 30')
So we get
= cosec 17^0 30'/ cosec 17^0 30'
= 1

2. (i) cot 40^0/tan 50^0 – ½ (cos 35^0/sin 55^0)
(ii) (sin 49^0/cos 41^0)^2 + (cos 41^0/sin 49^0)^2
(iii) sin 72^0/cos 18^0 – sec 32^0/cosec 58^0
(iv) cos 75^0/sin 15^0 + sin 12^0/ cos 78^0 – cos 18^0/sin 72^0
(v) sin 25^0/sec 65^0 + cos 25^0/ cosec 65^0.
Solution:

(i) cot 40^0/tan 50^0 – ½ (cos 35^0/sin 55^0)
It can be written as

$$= \frac{cot40^0}{tan(90^0 - 40^0)} - \frac{1}{2}\frac{cos35^0}{sin(90^0 - 35^0)}$$

$By\ further\ calculation$

$$= \frac{cot40^0}{cot40^0} - \frac{1}{2}\frac{cos35^0}{cos35^0}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

(ii) $(\sin 49^0/\cos 41^0)^2 + (\cos 41^0/\sin 49^0)^2$

It can be written as

$$= [\frac{sin49^0}{cos(90^0 - 49^0)}]^2 + [\frac{cos41^0}{sin(90^0 - 41^0)}]^2$$

$By\ further\ calculation$

$$= (\frac{sin49^0}{sin49^0})^2 + (\frac{cos41^0}{cos41^0})^2$$

So we get

$= 1^2 + 1^2$

= 1 + 1

= 2

(iii) $\sin 72^0/\cos 18^0 - \sec 32^0/\text{cosec } 58^0$

It can be written as

$$= \frac{sin72^0}{cos(90^0 - 72^0)} - \frac{sec32^0}{cosec(90^0 - 32^0)}$$

$By\ further\ calculation$

$$= \frac{sin72^0}{sin72^0} - \frac{sec32^0}{sec32^0}$$

= 1 – 1

= 0

(iv) $\cos 75^0/\sin 15^0 + \sin 12^0/ \cos 78^0 - \cos 18^0/\sin 72^0$

It can be written as

$$= \frac{cos75^0}{sin(90^0 - 75^0)} + \frac{sin12^0}{cos(90^0 - 12^0)} - \frac{cos18^0}{sin(90^0 - 18^0)}$$

$By\ further\ calculation$

$$= \frac{cos75^0}{cos75^0} + \frac{sin12^0}{sin12^0} - \frac{cos18^0}{cos18^0}$$

So we get
= 1 + 1 – 1
= 1

(v) sin 25^0/sec 65^0 + cos 25^0/ cosec 65^0
It can be written as
= (sin 25^0 × cos 65^0) + (cos 25^0 × sin 65^0)
By further calculation
= [sin 25^0 × cos (90^0 – 25^0)] + [cos 25^0 × sin (90^0 – 25^0)]
So we get
= (sin 25^0 × sin 25^0) + (cos 25^0 × cos 25^0)
= $\sin^2 25^0 + \cos^2 25^0$
= 1

3. (i) sin 62^0 – cos 28^0
(ii) cosec 35^0 – sec 55^0.
Solution:

(i) sin 62^0 – cos 28^0
It can be written as
= sin (90^0 – 28^0) – cos 28^0
So we get
= cos 28^0 – cos 28^0
= 0

(ii) cosec 35^0 – sec 55^0
It can be written as
= cosec 35^0 – sec (90^0 – 35^0)
So we get
= cosec 35^0 – cosec 35^0
= 0

4. (i) $\cos^2 26^0$ + cos 64^0 sin 26^0 + tan 36^0/ cot 54^0
(ii) sec 17^0/cosec 73^0 + tan 68^0/cot 22^0 + $\cos^2 44^0$ + $\cos^2 46^0$.
Solution:

(i) $\cos^2 26^0$ + cos 64^0 sin 26^0 + tan 36^0/ cot 54^0
It can be written as

$= \cos^2 26^0 + \cos (90^0 – 26^0) \sin 26^0 + \tan 36^0/\cot (90^0 – 36^0)$
We know that
$\cos (90^0 – \theta) = \sin \theta$
$\cot (90^0 – \theta) = \tan \theta$
$\sin^2 \theta + \cos^2 \theta = 1$
So we get
$= \cos^2 26^0 + \sin^2 26^0 + \tan 36^0/\tan 36^0$
$= 1 + 1$
$= 2$

(ii) $\sec 17^0/\text{cosec } 73^0 + \tan 68^0/\cot 22^0 + \cos^2 44^0 + \cos^2 46^0$
It can be written as
$= \sec (90^0 – 73^0)/ \text{cosec } 73^0 + \tan (90^0 – 22^0)/ \cot 22^0 + \cos^2 (90^0 – 46^0) + \cos^2 46^0$
By further calculation
$= \text{cosec } 73^0/ \text{cosec } 73^0 + \cot 22^0/ \cot 22^0 + \sin^2 46^0 + \cos^2 46^0$
We know that $\sin^2 \theta + \cos^2 \theta = 1$
So we get
$= 1 + 1 + 1$
$= 3$

5. (i) $\cos 65^0/\sin 25^0 + \cos 32^0/\sin 58^0 – \sin 28^0 \sec 62^0 + \text{cosec}^2 30^0$
(ii) $\sec 29^0/ \text{cosec } 61^0 + 2 \cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0 – 3 (\sin^2 38^0 + \sin^2 52^0)$.
Solution:

(i) $\cos 65^0/\sin 25^0 + \cos 32^0/\sin 58^0 – \sin 28^0 \sec 62^0 + \text{cosec}^2 30^0$
It can be written as
$= \cos 65^0/ \sin (90^0 – 65^0) + \cos 32^0/ \sin (90^0 – 32^0) – \sin 28^0 \sec (90^0 – 28^0) + \text{cosec}^2 30^0$
By further calculation
$= \cos 65^0/\cos 65^0 + \cos 32^0/ \cos 32^0 – \sin 28^0 \text{ cosec } 28^0 + \text{cosec}^2 30^0$
We know that $\text{cosec } 30^0 = 2$
$= 1 + 1 – 1 + 4$
$= 5$

(ii) $\sec 29^0/ \text{cosec } 61^0 + 2 \cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0 – 3 (\sin^2 38^0 + \sin^2 52^0)$
It can be written as
$= \sec 29^0/ \text{cosec } (90^0 - 29^0) + 2 \cot 8^0 \cot 17^0 \cot 45^0 \cot (90^0 – 17^0) \cot (90^0 – 8^0) – 3 [\sin^2 38^0 + \sin^2 (90^0 - 38^0)]$
By further calculation
$= \sec 29^0/ \sec 29^0 + 2 \cot 8^0 \cot 17^0 \times 1 \times \tan 17^0 \tan 8^0 – 3 (\sin^2 38^0 + \cos^2 38^0)$
So we get
$= 1 + 2 \cot 8^0 \tan 8^0 \cot 17^0 \tan 17^0 \times 1 – 3 \times 1$
We know that
$\text{cosec } (90^0 – \theta) = \sec \theta$
$\cot (90^0 – \theta) = \tan \theta$
$\sin^2 \theta + \cos^2 \theta = 1$
Here

$= 1 + 2 \times 1 \times 1 \times 1 - 3$
$= 1 + 2 - 3$
$= 0$

6. Express each of the following in terms of trigonometric ratios of angles between 0^0 to 45^0:
(i) $\tan 81^0 + \cos 72^0$
(ii) $\cot 49^0 + \text{cosec } 87^0$.
Solution:

(i) $\tan 81^0 + \cos 72^0$
It can be written as
$= \tan (90^0 - 9^0) + \cos (90^0 - 18^0)$
So we get
$= \cot 9^0 + \sin 18^0$

(ii) $\cot 49^0 + \text{cosec } 87^0$
It can be written as
$= \cot (90^0 - 41^0) + \text{cosec } (90^0 - 3^0)$
So we get
$= \tan 41^0 + \sec 3^0$

Without using trigonometric tables, prove that (7 to 11):
7. (i) $\sin^2 28^0 - \cos^2 62^0 = 0$
(ii) $\cos^2 25^0 + \cos^2 65^0 = 1$
(iii) $\text{cosec}^2 67^0 - \tan^2 23^0 = 1$
(iv) $\sec^2 22^0 - \cot^2 68^0 = 1$.
Solution:

(i) $\sin^2 28^0 - \cos^2 62^0 = 0$
Consider
LHS $= \sin^2 28^0 - \cos^2 62^0$
It can be written as
$= \sin^2 28^0 - \cos^2 (90^0 - 28^0)$
So we get
$= \sin^2 28^0 - \sin^2 28^0$
$= 0$
= RHS

(ii) $\cos^2 25^0 + \cos^2 65^0 = 1$
Consider
LHS $= \cos^2 25^0 + \cos^2 65^0$
It can be written as
$= \cos^2 25^0 + \cos^2 (90^0 - 25^0)$
We know that $\sin^2 \theta + \cos^2 \theta = 1$
So we get
$= \cos^2 25^0 + \sin^2 25^0$

= 1

(iii) $\text{cosec}^2 67^0 - \tan^2 23^0 = 1$
Consider
LHS = $\text{cosec}^2 67^0 - \tan^2 23^0$
It can be written as
= $\text{cosec}^2 67^0 - \tan^2 (90^0 - 67^0)$
We know that $\text{cosec}^2 \theta - \cot^2 \theta = 1$
So we get
= $\text{cosec}^2 67^0 - \cot^2 67^0$
= 1

(iv) $\sec^2 22^0 - \cot^2 68^0 = 1$
Consider
LHS = $\sec^2 22^0 - \cot^2 68^0$
It can be written as
= $\sec^2 22^0 - \cot^2 (90^0 - 22^0)$
We know that $\sec^2 \theta - \tan^2 \theta = 1$
So we get
= $\sec^2 22^0 - \tan^2 22^0$
= 1

8. (i) $\sin 63^0 \cos 27^0 + \cos 63^0 \sin 27^0 = 1$
(ii) $\sec 31^0 \sin 59^0 + \cos 31^0 \text{cosec } 59^0 = 2$.
Solution:

(i) $\sin 63^0 \cos 27^0 + \cos 63^0 \sin 27^0 = 1$
Consider
LHS = $\sin 63^0 \cos 27^0 + \cos 63^0 \sin 27^0$
It can be written as
= $\sin 63^0 \cos (90^0 - 63^0) + \cos 63^0 \sin (90^0 - 63^0)$
= $\sin 63^0 \sin 63^0 + \cos 63^0 \cos 63^0$
We know that $\sin^2 \theta + \cos^2 \theta = 1$
So we get
= $\sin^2 63^0 + \cos^2 63^0$
= 1

(ii) $\sec 31^0 \sin 59^0 + \cos 31^0 \text{cosec } 59^0 = 2$
Consider
LHS = $\sec 31^0 \sin 59^0 + \cos 31^0 \text{cosec } 59^0$
It can be written as
= $1/ \cos 31^0 \times \sin 59^0 + \cos 31^0 \times 1/ \sin 59^0$
By further calculation
= $\sin 59^0/\cos (90^0 - 59^0) + \cos 31^0/ \sin (90^0 - 31^0)$
So we get
= $\sin 59^0/\sin 59^0 + \cos 31^0/\cos 31^0$

= 1 + 1
= 2
= RHS

9. (i) sec 70^0 sin 20^0 – cos 20^0 cosec 70^0 = 0
(ii) $\sin^2 20^0 + \sin^2 70^0 - \tan^2 45^0 = 0$.
Solution:

(i) sec 70^0 sin 20^0 – cos 20^0 cosec 70^0 = 0
Consider
LHS = sec 70^0 sin 20^0 – cos 20^0 cosec 70^0
By further simplification
= sin 20^0/ cos 70^0 – cos 20^0/ sin 70^0
It can be written as
= sin 20^0/ cos $(90^0 - 20^0)$ – cos 20^0/ sin $(90^0 - 20^0)$
So we get
= sin 20^0/ sin 20^0 – cos 20^0/ cos 20^0
= 1 – 1
= 0
= RHS

(ii) $\sin^2 20^0 + \sin^2 70^0 - \tan^2 45^0 = 0$
Consider
LHS = $\sin^2 20^0 + \sin^2 70^0 - \tan^2 45^0$
It can be written as
= $\sin^2 20^0 + \sin^2 (90^0 - 20^0) - \tan^2 45^0$
By further calculation
= $\sin^2 20^0 + \cos^2 20^0 - \tan^2 45^0$
We know that $\sin^2 \theta + \cos^2 \theta = 1$ and tan 45^0 = 1
So we get
= 1 – 1
= 0
= RHS

10. (i) cot 54^0/tan 36^0 + tan 20^0/cot 70^0 – 2 = 0
(ii) sin 50^0/cos 40^0 + cosec 40^0/sec 50^0 – 4 cos 50^0 cosec 40^0 + 2 = 0.
Solution:

(i) cot 54^0/tan 36^0 + tan 20^0/cot 70^0 – 2 = 0
Consider
LHS = cot 54^0/tan 36^0 + tan 20^0/cot 70^0 – 2
It can be written as
= cot 54^0/tan $(90^0 - 54^0)$ + tan 20^0/cot $(90^0 - 20^0)$ – 2
By further calculation
= cot 54^0/ cot 54^0 + tan 20^0/ tan 20^0 – 2
So we get

= 1 + 1 - 2
= 0
= RHS

(ii) $\sin 50^0/\cos 40^0 + \text{cosec } 40^0/\sec 50^0 – 4 \cos 50^0 \text{ cosec } 40^0 + 2 = 0$
Consider
LHS = $\sin 50^0/\cos 40^0 + \text{cosec } 40^0/\sec 50^0 – 4 \cos 50^0 \text{ cosec } 40^0 + 2$
It can be written as
= $\sin 50^0/\cos (90^0 – 50^0) + \text{cosec } 40^0/\sec (90^0 – 40^0) – 4 \cos 50^0 \text{ cosec } (90^0 – 50^0) + 2$
By further calculation
= $\sin 50^0/\sin 50^0 + \text{cosec } 40^0/\text{cosec } 40^0 – \cos 50^0 \sec 50^0 + 2$
So we get
= $1 + 1 – 4 \cos 50^0/\cos 50^0 + 2$
= 1 + 1 – 4 + 2
= 4 – 4
= 0
= RHS

11. (i) $\cos 70^0/\sin 20^0 + \cos 59^0/\sin 31^0 – 8 \sin^2 30^0 = 0$
(ii) $\cos 80^0/\sin 10^0 + \cos 59^0 \text{ cosec } 31^0 = 2$.
Solution:

(i) $\cos 70^0/\sin 20^0 + \cos 59^0/\sin 31^0 – 8 \sin^2 30^0 = 0$
Consider
LHS = $\cos 70^0/\sin 20^0 + \cos 59^0/\sin 31^0 – 8 \sin^2 30^0$
It can be written as
= $\cos 70^0/\sin (90^0 – 70^0) + \cos 59^0/\sin (90^0 – 59^0) – 8 \sin^2 30^0$
We know that $\sin 30^0 = ½$
= $\cos 70^0/\cos 70^0 + \cos 59^0/ \cos 59^0 – 8 (1/2)^2$
By further calculation
= 1 + 1 – 8 × ¼
So we get
= 2 – 2
= 0
= RHS

(ii) $\cos 80^0/\sin 10^0 + \cos 59^0 \text{ cosec } 31^0 = 2$
Consider
LHS = $\cos 80^0/\sin 10^0 + \cos 59^0 \text{ cosec } 31^0$
It can be written as
= $\cos 80^0/\sin (90^0 – 80^0) + \cos 59^0/ \sin 31^0$
By further simplification
= $\cos 80^0/ \cos 80^0 + \cos 59^0/ \sin (90^0 – 59^0)$
So we get
= $1 + \cos 59^0/ \cos 59^0$
= 1 + 1

= 2
= RHS

12. Without using trigonometrical tables, evaluate:

$$(i) 2(\frac{tan35^0}{cot55^0})^2 + (\frac{cot55^0}{tan35^0}) - 3(\frac{sec40^0}{cosec50^0})$$

$$(ii) \frac{sin35^0 cos55^0 + cos35^0 sin55^0}{cosec^2 10^0 - tan^2 80^0}$$

(iii) $\sin^2 34^0 + \sin^2 56^0 + 2 \tan 18^0 \tan 72^0 - \cot^2 30^0$.

Solution:

$$(i) 2(\frac{tan35^0}{cot55^0})^2 + (\frac{cot55^0}{tan35^0}) - 3(\frac{sec40^0}{cosec50^0})$$

$It\ can\ be\ written\ as$

$$= 2[\frac{tan(90^0 - 55^0)}{cot55^0}] + [\frac{cot(90^0 - 35^0)}{tan35^0}] - 3[\frac{sec(90^0 - 50^0)}{cosec50^0}]$$

$By\ further\ calculation$

$$= 2(\frac{cot55^0}{cot55^0})^2 + (\frac{tan35^0}{tan35^0}) - 3(\frac{cosec50^0}{cosec50^0})$$

So we get
= 2 + 1 – 3
= 0

$$(ii) \frac{sin35^0 cos55^0 + cos35^0 sin55^0}{cosec^2 10^0 - tan^2 80^0}$$

$It\ can\ be\ written\ as$

$$= \frac{sin35^0 cos(90^0 - 35^0) + cos35^0 sin(90^0 - 35^0)}{cosec^2 10^0 - tan^2 (90^0 - 10^0)}$$

$By\ further\ calculation$

$$= \frac{sin35^0 . sin35^0 + cos35^0 . cos35^0}{cosec^2 10^0 - cot^2 10^0}$$

$So\ we\ get$

$$= \frac{sin^2 35^0 + cos^2 35^0}{cosec^2 10^0 - cot^2 10^0}$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$ and $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
= 1/1
= 1

(iii) $\sin^2 34^0 + \sin^2 56^0 + 2 \tan 18^0 \tan 72^0 - \cot^2 30^0$
It can be written as
$= \sin^2 34^0 + [\sin (90^0 - 34^0)]^2 + 2 \tan 18^0 \tan (90^0 - 18^0) - \cot^2 30^0$
By further simplification
$= \sin^2 34^0 + \cos^2 34^0 + 2 \tan 18^0 \cot 18^0 - (\sqrt{3})^2$
So we get
$= 1 + 2 \tan 18^0 \times 1/\tan 18^0 - 3$
$= 1 + 2 - 3$
$= 0$

13. Prove the following:

$$(i) \frac{cos\Theta}{sin(90^0-\Theta)} + \frac{sin\Theta}{cos(90^0-\Theta)} = 2$$

$$(ii) cos\Theta sin(90^0-\Theta) + sin\Theta cos(90^0-\Theta) = 1$$

$$(iii) \frac{tan\Theta}{tan(90^0-\Theta)} + \frac{sin(90^0-\Theta)}{cos\Theta} = sec^2\Theta.$$

Solution:

$$(i) \frac{cos\Theta}{sin(90^0-\Theta)} + \frac{sin\Theta}{cos(90^0-\Theta)} = 2$$

We know that

LHS = $\frac{cos\Theta}{sin(90^0-\Theta)} + \frac{sin\Theta}{cos(90^0-\Theta)}$

So we get
$= \cos \theta/ \cos \theta + \sin \theta/ \sin \theta$
$= 1 + 1$
$= 2$
= RHS

(ii) $\cos \theta \sin (90^0 - \theta) + \sin \theta \cos (90^0 - \theta) = 1$
Consider
LHS = $\cos \theta \sin (90^0 - \theta) + \sin \theta \cos (90^0 - \theta)$
It can be written as
$= \cos \theta. \cos \theta + \sin \theta. \sin \theta$
So we get
$= \cos^2 \theta + \sin^2 \theta$
$= 1$
= RHS

$$(iii)\frac{tan\Theta}{tan(90^0-\Theta)}+\frac{sin(90^0-\Theta)}{cos\Theta}=sec^2\Theta$$

Consider

LHS = $\frac{tan\Theta}{tan(90^0-\Theta)}+\frac{sin(90^0-\Theta)}{cos\Theta}$

By further calculation

= tan θ/ cot θ + cos θ/ cos θ

So we get

= tan θ × tan θ + 1

= $\tan^2$ θ + 1

= $\sec^2$ θ

= RHS

14. Prove the following:

$$(i)\frac{cos(90^0-A)sin(90^0-A)}{tan(90^0-A)}=1-cos^2A$$

$$(ii)\frac{sin(90^0-A)}{cosec(90^0-A)}+\frac{cos(90^0-A)}{sec(90^0-A)}=1.$$

Solution:

$$(i)\frac{cos(90^0-A)sin(90^0-A)}{tan(90^0-A)}=1-cos^2A$$

Consider

LHS = $\frac{cos(90^0-A)sin(90^0-A)}{tan(90^0-A)}$

It can be written as

= sin A cos A/ cot A

So we get

= (sin A cos A × sin A)/ cos A

= $\sin^2$ A

= 1 – $\cos^2$ A

= RHS

$$(ii)\frac{sin(90^0-A)}{cosec(90^0-A)}+\frac{cos(90^0-A)}{sec(90^0-A)}=1$$

Consider

LHS = $\frac{sin(90^0-A)}{cosec(90^0-A)}+\frac{cos(90^0-A)}{sec(90^0-A)}$

It can be written as

= cos A/ sec A + sin A/ cosec A
So we get
= cos A × cos A + sin A × sin A
= $\cos^2$ A + $\sin^2$ A
= 1
= RHS

15. Simplify the following:

$$(i)\frac{cos\Theta}{sin(90^0-\Theta)}+\frac{cos(90^0-\Theta)}{sec(90^0-\Theta)}-3tan^2 30^0$$

$$(ii)\frac{cosec(90^0-\Theta)sin(90^0-\Theta)cot(90^0-\Theta)}{cos(90^0-\Theta)sec(90^0-\Theta)tan\Theta}+\frac{cot\Theta}{tan(90^0-\Theta)}.$$

Solution:

$$(i)\frac{cos\Theta}{sin(90^0-\Theta)}+\frac{cos(90^0-\Theta)}{sec(90^0-\Theta)}-3tan^2 30^0$$

It can be written as
= cos θ/ cos θ + sin θ/ cosec θ – 3 $\tan^2 30^0$
By further calculation
= 1 + sin θ × sin θ – 3 $(1/ \sqrt{3})^2$
So we get
= $\sin^2$ θ + 1 – 3 × 1/3
= $\sin^2$ θ + 1 – 1
= $\sin^2$ θ +

$$(ii)\frac{cosec(90^0-\Theta)sin(90^0-\Theta)cot(90^0-\Theta)}{cos(90^0-\Theta)sec(90^0-\Theta)tan\Theta}+\frac{cot\Theta}{tan(90^0-\Theta)}$$

It can be written as
= (sec θ cos θ tan θ)/ (sin θ cosec θ tan θ) + cot θ/ cot θ
So we get
= sec θ cos θ/ sin θ cosec θ + 1
= 1/1 + 1
= 1 + 1
= 2

16. Show that

$$\frac{cos^2(45^0+\Theta)+cos^2(45^0-\Theta)}{tan(60^0+\Theta)tan(30^0-\Theta)}=1.$$

Solution:

Consider

$$LHS = \frac{cos^2(45^0+\Theta)+cos^2(45^0-\Theta)}{tan(60^0+\Theta)tan(30^0-\Theta)}$$

$It\ can\ be\ written\ as$

$$= \frac{cos^2(45^0+\Theta)+cos^2[90^0-(45^0-\Theta)]}{tan(60^0+\Theta)tan[90^0-(60^0-\Theta)]}$$

$By\ further\ calculation$

$$= \frac{cos^2(45^0+\Theta)+sin^2(45^0-\Theta)}{tan(60^0+\Theta)cot(60^0-\Theta)}$$

We know that cos (90^0 – θ) = sin θ, tan (90^0 – θ) = cot θ and tan θ cot θ = 1
So we get
= 1/1
= 1
= RHS

17. Find the value of A if
(i) sin 3A = cos (A – 6^0), where 3A and A – 6^0 are acute angles
(ii) tan 2A = cot (A – 18^0), where 2A and A – 18^0 are acute angles
(iii) If sec 2A = cosec (A – 27^0) where 2A is an acute angle, find the measure of ∠A.
Solution:

(i) sin 3A = cos (A – 6^0), where 3A and A – 6^0 are acute angles
It is given that
sin 3A = cos (A – 6^0)
We know that cos (90^0 – θ) = sin θ
cos (90^0 – 3A) = cos (A – 6^0)
By comparing both
90^0 – 3A = A – 6^0
By further calculation
90^0 + 6^0 = A + 3A
96^0 = 4A
So we get
A = 96^0/4 = 24^0

Hence, the value of A is 24^0.

(ii) tan 2A = cot (A – 18^0)
We know that cot (90^0 – θ) = tan θ
cot (90^0 – 2A) = cot (A – 18^0)
By comparing both
90^0 – 2A = A – 18^0
By further calculation

$90^0 + 18^0 = A + 2A$
So we get
$3A = 108^0$
$A = 108^0/3 = 36^0$

Hence, the value of A is 36^0.

(iii) sec 2A = cosec (A – 27^0)
We know that cosec (90^0 - θ) = sec θ
cosec (90^0 – 2A) = cos (A – 27^0)
By comparing both
$90^0 - 2A = A - 27^0$
By further calculation
$90^0 + 27^0 = A + 2A$
So we get
$3A = 117^0$
$A = 117^0/3 = 39^0$

Hence, the value of A is 39^0.

18. Find the value of θ ($0^0 < \theta < 90^0$) if:
(i) cos 63^0 sec (90^0 – θ) = 1
(ii) tan 35^0 cot (90^0 – θ) = 1.
Solution:

(i) cos 63^0 sec (90^0 – θ) = 1
It can be written as
cos 63^0 = 1/sec (90^0 - θ)
We know that 1/sec θ = cos θ
cos 63^0 = cos (90^0 - θ)
By comparing both
$90^0 - \theta = 63^0$
By further calculation
$\theta = 90^0 - 63^0 = 27^0$

(ii) tan 35^0 cot (90^0 – θ) = 1
It can be written as
tan 35^0 = 1/cot (90^0 - θ)
We know that 1/cot θ = cos θ
tan 35^0 = tan (90^0 - θ)
By comparing both
$35^0 = 90^0 - \theta$
By further calculation
$\theta = 90^0 - 35^0 = 55^0$

19. If A, B and C are the interior angles of a △ ABC, show that

(i) cos (A + B)/2 = sin C/2
(ii) tan (C + A)/2 = cot B/2.
Solution:

A, B and C are the interior angles of a △ ABC
It can be written as
$\angle A + \angle B + \angle C = 180^0$
Dividing both sides by 2
$(\angle A + \angle B + \angle C)/2 = 180^0/2$
$A/2 + B/2 + C/2 = 90^0$

(i) cos (A + B)/2 = sin C/2
We can write it as
$(A + B)/2 = 90^0 – C/2$
We know that
$\cos (90^0 – C/2) = \sin C/2$
Here $\cos (90^0 – \theta) = \sin \theta$
sin C/2 = sin C/2

(ii) tan (C + A)/2 = cot B/2
We know that $(A + C)/2 = 90^0 – B/2$
$= \tan (90^0 – B/2)$
So we get
= cot B/2
= RHS

CHAPTER TEST

1. Find the values of:

(i) $\sin^2 60° - \cos^2 45° + 3\tan^2 30°$

(ii) $\dfrac{2\cos^2 45° + 3\tan^2 30°}{\sqrt{3}\cos 30° + \sin 30°}$

(iii) sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60°

Solution:

(i) $\sin^2 60° - \cos^2 45° + 3\tan^2 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} - \frac{1}{2} + 3 \times \frac{1}{3} = \frac{3}{4} - \frac{1}{2} + \frac{1}{1}$$

$$= \frac{3-2+4}{4} = \frac{7-2}{4} = \frac{5}{4} = 1\frac{1}{4}$$

Therefore, $\sin^2 60° - \cos^2 45° + 3\tan^2 30° = 1\frac{1}{4}$

(ii) $\dfrac{2\cos^2 45° + 3\tan^2 30°}{\sqrt{3}\cos 45° + \sin 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{1}{\sqrt{3}}\right)^2}{\sqrt{3} \times \left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}}$$

$$= \frac{2 \times \frac{1}{2} + 3 \times \frac{1}{3}}{\frac{3}{2} + \frac{1}{2}} = \frac{1+1}{\frac{3+1}{2}} = \frac{2}{\frac{4}{2}} = \frac{2}{2} = 1$$

Hence, $\dfrac{2\cos^2 45° + 3\tan^2 30°}{\sqrt{3}\cos 30° + \sin 30°} = 1$

(iii) sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60°

$$= \frac{2}{\sqrt{3}} \times \sqrt{3} + \frac{1}{\sqrt{2}} \times \sqrt{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{2}{1} + \frac{1}{1} + \frac{1}{2}$$

$= 2 + 1 + ½ = 3 + ½ = (6 + 1)/2$

= 7/2 = 3½

Thus, sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60° = 3½

2. Taking A = 30°, verify that

(i) $\cos^4 A - \sin^4 A = \cos 2A$

(ii) 4cos A cos (60° – A) cos (60° + A) = cos 3 A.

Solution:

(i) $\cos^4 A - \sin^4 A = \cos 2A$

Let's take A = 30°

so, we have

L.H.S.= $\cos^4 A - \sin^4 A = \cos^4 30° - \sin^4 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$$

$$= \frac{\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}}{2\times2\times2\times2} - \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{9}{16} - \frac{1}{16}$$

$$= \frac{9-1}{16} = \frac{8}{16} = \frac{1}{2}$$

Now,

R.H.S. = **c**os 2A = cos 2(30º) = ½

Therefore, L.H.S. = R.H.S. hence verified.

(ii) 4 cos A cos (60°- A) cos (60° + A) = cos 3 A

Let's take A = 30°

L.H.S. = 4 cos A cos (60° – A) cos (60° + A)

= 4 cos 30° cos (60° – 30°) cos (60° + 30°)

= 4 cos 30° cos 30° cos 90°

= 4 × (√3/2) × (√3/2) × 0

= 0

Now,

R.H.S. = cos 3A

= cos (3 × 30°) = cos 90° = 0

Hence, L.H.S. = R.H.S. hence verified.

3. If A = 45° and B = 30°, verify that sin A/ (cos A + sin A + sin B) = 2/3

Solution:

Taking,

$$\text{L.H.S. } \frac{\sin A}{\cos A + \sin A \sin B}$$
$$= \frac{\sin 45^\circ}{\cos 45^\circ + \sin 45^\circ \sin 30^\circ}$$
$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}}$$
$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2\sqrt{2} + \sqrt{2}}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{4}}$$
$$= \frac{\sqrt{2}}{2} \times \frac{4}{3\sqrt{2}} = \frac{2}{3} = \text{R.H.S.}$$

Hence verified.

4. Taking A = 60° and B = 30°, verify that
(i) sin (A + B)/ cos A cos B = tan A + tan B
(ii) sin (A - B)/ sin A sin B = cot B – cot A
Solution:

(i) Here, A = 60° and B = 30°

$$\text{LHS} = \frac{\sin(A+B)}{\cos A \cos B} = \frac{\sin(60^\circ + 30^\circ)}{\cos 60^\circ \cos 30^\circ}$$
$$= \frac{\sin 90^\circ}{\cos 60^\circ \cos 30^\circ} = \frac{1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}}$$
$$= \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$$

Now,

$$\text{R.H.S.} = \tan A + \tan B$$
$$= \tan 60^\circ + \tan 30^\circ$$
$$= \sqrt{3} + \frac{1}{\sqrt{3}}$$
$$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$\therefore$ L.H.S. = R.H.S.

(ii) A = 60°, B = 30°

$$\text{L.H.S.} = \frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin(60^\circ - 30^\circ)}{\sin 60^\circ \sin 30^\circ}$$

$$= \frac{\sin 30^\circ}{\sin 60^\circ \sin 30^\circ} = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

$$\text{R.H.S.} = \cot B - \cot A$$

$$= \cot 30^\circ - \cot 60^\circ$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$\therefore$ L.H.S. = R.H.S.

5. If √2 tan 2θ = √6 and θ° < 2θ < 90°, find the value of sin θ + √3 cos θ – 2 tan^2 θ.
Solution:

Given,
√2 tan 2θ = √6
tan 2θ = √6/ √2
= √3
= tan 60°
⇒ 2θ = 60°
θ = 30°
Now,
sin θ + √3 cos θ – 2 tan^2 θ
= sin 30° + √3 cos 30° – 2 tan^2 30°
= ½ + √3 x √3/2 – 2 (1/√3)^2
= ½ + 3/2 – 2/3
= 4/2 – 2/3
= (12 - 4)/6
= 8/6
= 4/3

6. If 3θ is an acute angle, solve the following equations for θ:
(i) (cosec 3θ – 2) (cot 2θ – 1) = 0
(ii) (tan θ – 1) (cosec 3θ – 1) = 0
Solution:

(i) (cosec 3θ – 2) (cot 2θ – 1) = 0

Now, either
cosec 3θ – 2 or cot 2θ – 1 = 0
⇒ cosec 3θ = 2 or cot 2θ = 1
So,
cosec 3θ = cosec 3θ° or cot 2θ =cot 45°
⇒ 3θ = 30° or 2θ = 45°
Thus, θ = 30° or 45°.

(ii) (tan θ – 1) (cosec 3θ – 1) = 0
Now, either
tan θ – 1 = 0 or cosec 3θ – 1 = 0
⇒ tan θ = 1 or cosec 3θ = 1
So,
tan θ = tan45° or cosec 3 θ = cosec 90°
⇒ θ = 45° or 3θ = 90° i.e. θ = 30°
Thus, θ = 45° or 30°.

7. If tan (A + B) = √3 and tan (A – B) = 1 and A, B (B < A) are acute angles, find the values of A and B.
Solution:

Given, tan (A + B) = √3
So, tan (A + B) = tan 60°   [Since, tan 60° = √3]
⇒ A + B = 60° ……….. (i)
Also, given
tan (A – B) = 1
So, tan (A – B) = tan 45° [tan 45° = 1]
⇒ A – B = 45° ………….. (ii)
From equation (1) and (2), we get
A + B = 60°
A – B = 45°

2A = 105°
A = 52½°
Now, on substituting the value of A in equation (i), we get
52½° + B = 60°
B = 60° - 52½° = 7½°
Therefore, the value of A = 52½° and B = 7½°

8. Without using trigonometrical tables, evaluate the following:
(i) $\sin^2 28° + \sin^2 62° – \tan^2 45°$
(ii) $2\frac{\cos 27°}{\sin 63°} + \frac{\tan 27°}{\cot 63°} + \cos 0°$
(iii) cos 18° sin 72° + sin 18° cos 72°
(iv) 5 sin 50° sec 40° – 3 cos 59° cosec 31°

Solution:

(i) $\sin^2 28° + \sin^2 62° – \tan^2 45°$
$= \sin^2 28° + \sin^2 (90° – 28°) – \tan^2 45°$
$= \sin^2 28° + \cos^2 28° – \tan^2 45°$
$= 1 – (1)^2$ $(\because \sin^2 \theta + \cos^2 \theta = 1$ and $\tan 45° = 1)$
$= 1 – 1$
$= 0$

(ii) $2\dfrac{\cos 27°}{\sin 63°} + \dfrac{\tan 27°}{\cot 63°} + \cos 0°$
$= 2\dfrac{\cos 27°}{\sin(90° - 27°)} + \dfrac{\tan 27°}{\cot(90° - 27°)} + \cos 0°$

$= 2\dfrac{\cos 27°}{\cos 27°} + \dfrac{\tan 27°}{\tan 27°} + 1$ $(\because \cos 0° = 1)$
$= 2 \times 1 + 1 + 1$
$= 2 + 1 + 1$
$= 4$

(iii) $\cos 18° \sin 12° + \sin 18° \cos 12°$
$= \cos (90° – 12°) \sin 72° + \sin (90° – 12°) \cos 12°$
$= \sin 72°.\sin 12° + \cos 12° \cos 12°$
$= \sin^2 12° + \cos^2 12°$
$= 1$ $(\because \sin^2 \theta + \cos^2 \theta = 1)$

(iv) $5 \sin 50° \sec 40° – 3 \cos 59° \operatorname{cosec} 31°$
$= 5\dfrac{\sin 50°}{\cos 40°} - 3\dfrac{\cos 59°}{\sin 31°}$
$= 5\dfrac{\sin 50°}{\cos(90° - 50°)} - 3\dfrac{\cos 59°}{\sin(90° - 59°)}$
$= 5\dfrac{\sin 50°}{\sin 50°} - 3\dfrac{\cos 59°}{\cos 59°} = 5 \times 1 - 3 \times 1$
$= 5 – 3$
$= 2$

9. Prove that:

$$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\operatorname{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta} = 2$$

Solution:

$$\text{L.H.S.} = \frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\operatorname{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta}$$

$$= \frac{\sin\theta\operatorname{cosec}\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1\times\tan\theta}{1\times\tan\theta} + 1 = 1 + 1 = 2 = \text{R.H.S.}$$

Thus, L.H.S. = R.H.S.
Hence proved.

10. When 0° < A < 90°, solve the following equations:
(i) sin 3A = cos 2A
(ii) tan 5A = cot A
Solution:

(i) sin 3A = cos 2A
⇒ sin 3A = sin (90° – 2A)
So,
3A = 90° – 2A
3 A + 2A = 90°
5A = 90°
∴ A = 90°/5 = 18°

(ii) tan 5A = cot A
⇒ tan 5A = tan (90° – A)
So,
5A = 90°- A
5A + A = 90°
6A = 90°
∴ A = 90°/6 = 15°

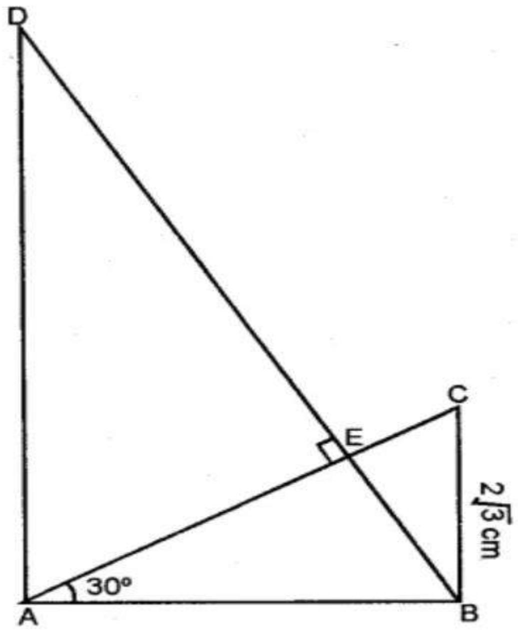
11. Find the value of θ if
(i) sin (θ + 36°) = cos θ, where θ and θ + 36° are acute angles.
(ii) sec 4θ = cosec (θ – 20°), where 4θ and θ – 20° are acute angles.
Solution:

(i) Given, θ and (θ + 36°) are acute angles
And,
sin (θ + 36°) = cos θ = sin (90° – θ) [As, sin (90° – θ) = cos θ]
On comparing, we get
θ + 36° = 90° – θ
θ + θ = 90° – 36°
2θ = 54°

$\theta = 54°/2$
$\therefore \theta = 27°$

(ii) Given, θ and $(\theta - 20°)$ are acute angles
And,
$\sec 4\theta = \text{cosec}\,(\theta - 20°)$
$\text{cosec}\,(90° - 4\theta) = \text{cosec}\,(\theta - 20°)$ [Since, $\text{cosec}\,(90° - \theta) = \sec\theta$]
On comparing, we get
$90° - 4\theta = \theta - 20°$
$90° + 20° = \theta + 4\theta$
$5\theta = 110°$
$\theta = 110°/5$
$\therefore \theta = 22°$

12. In the adjoining figure, ABC is right-angled triangle at B and ABD is right angled triangle at A. If BD ⊥ AC and BC = 2√3cm, find the length of AD.



Solution:

Given, ΔABC and ΔABD are right angled triangles in which ∠A = 90° and ∠B = 90°
And,
BC = $2\sqrt{3}$ cm. AC and BD intersect each other at E at right angle and ∠CAB = 30°.
Now in right ΔABC, we have
$\tan\theta = BC/AB$
$\Rightarrow \tan 30° = 2\sqrt{3}/ AB$
$\Rightarrow 1/\sqrt{3} = 2\sqrt{3}/ AB$
$\Rightarrow AB = 2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$ cm.
In ΔABE, ∠EAB = 30° and ∠EAB = 90°
Hence,

$\angle ABE$ or $\angle ABD = 180° - 90° - 30°$

$= 60°$

Now in right ΔABD, we have

$\tan 60° = AD/AB$

$\Rightarrow \sqrt{3} = AD/6$

Thus, $AD = 6\sqrt{3}$ cm.