EXERCISE 19.1

1. Find the co-ordinates of points whose
(i) abscissa is 3 and ordinate -4.
(ii)abscissa is -3/2 and ordinate 5.
$(iii)\, whose\ abscissa\ is\ -1\frac{2}{3}\ and\ ordinate\ is -2\frac{1}{4}.$
(iv) whose ordinate is 5 and abscissa is -2
(v) whose abscissa is -2 and lies on x-axis.
(vi) whose ordinate is 3/2 and lies on y-axis.

Solution:
Abscissa is the x-coordinate and ordinate is the y-coordinate of a point.
(i)The coordinate of the point whose abscissa is 3 and ordinate is -4 is (3,-4).
(ii)The coordinate of the point whose abscissa is -3/2 and ordinate is 5 is (-3/2,5).
(iii) The coordinate of the point whose
$abscissa\ is -1\frac{2}{3}\ and\ ordinate\ is -2\frac{1}{4}\ is\ \left(-1\frac{2}{3}, -2\frac{1}{4}\right)$.
(iv) The coordinate of the point whose ordinate is 5 and abscissa is -2 is (-2,5).
(v) The coordinate of the point whose abscissa is -2 and lies on x-axis is (-2,0).
If a point lies on x-axis, its y-coordinate is zero.
(vi) The coordinate of the point whose ordinate is 3/2 and lies on y-axis is (0,3/2).
If a point lies on y-axis, its x-coordinate is zero.

2. In which quadrant or on which axis each of the following points lie?
(-3, 5), (4, -1) (2, 0), (2, 2), (-3, -6)

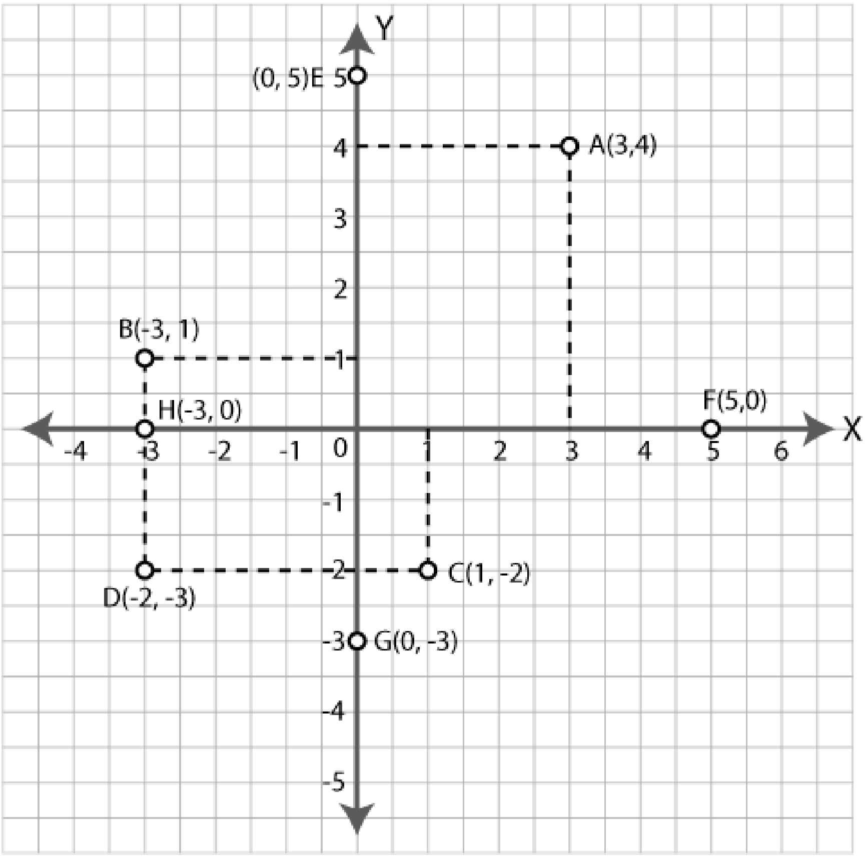
Solution:
In first quadrant, both x and y coordinate are positive.
In second quadrant, x-coordinate is negative and y-coordinate is positive.
In third quadrant, x-coordinate is negative and y-coordinate is negative.
In fourth quadrant, x-coordinate is positive and y-coordinate is negative.
(-3,5) lies in second quadrant.
(4,-1) lies in fourth quadrant.
(2,0) lies on x-axis. Here y-coordinate is zero.
(2,2) lies in first quadrant.
(-3,-6) lies in third quadrant.

3. Which of the following points lie on
(i) x-axis? (ii) y-axis?
A (0, 2), B (5, 6), C (23, 0), D (0, 23), E (0, -4), F (-6, 0), G (√3,0)

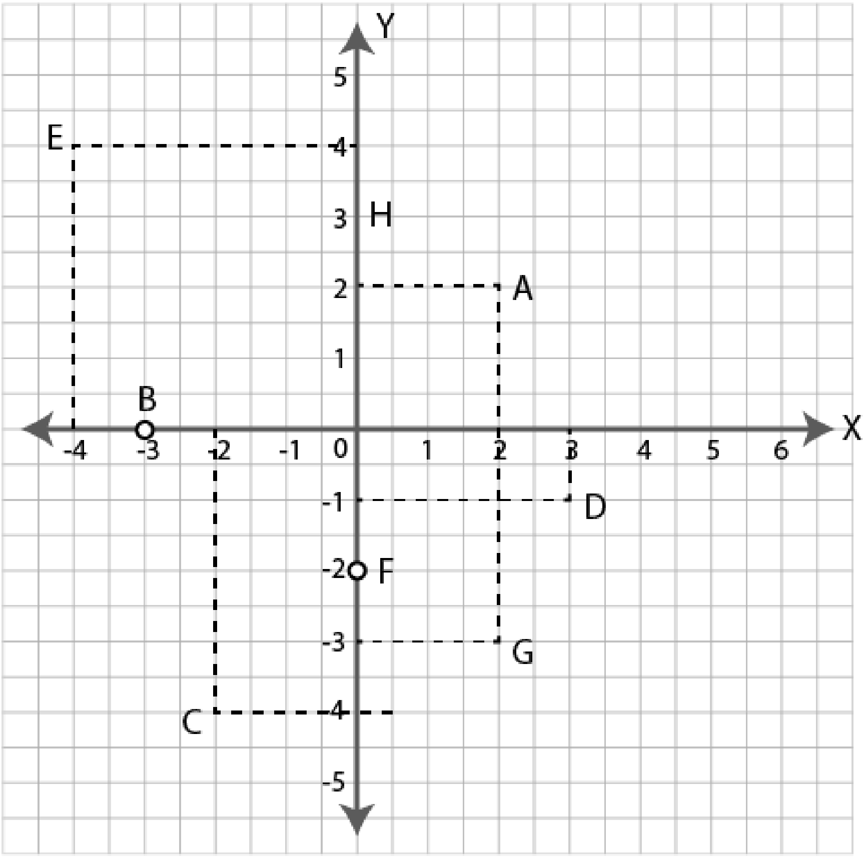
Solution:
Given points are A (0, 2), B (5, 6), C (23, 0), D (0, 23), E (0, -4), F (-6, 0), G (√3,0)
(i) If y-coordinate of a point is zero, then the point lies on X-axis.
So C(23,0), F(-6,0) and G(√3,0) lies X-axis.
(ii) If x-coordinate of a point is zero, then the point lies on Y-axis.
So A(0,2), D(0,23) and E(0,-4) lies Y-axis.

4. Plot the following points on the same graph paper :
A (3, 4), B (-3, 1), C (1, -2), D (-2, -3), E (0, 5), F (5, 0), G (0, -3), H (-3, 0).

Solution:
Given points are A (3, 4), B (-3, 1), C (1, -2), D (-2, -3), E (0, 5), F (5, 0), G (0, -3), H (-3, 0).
The points are plotted in the graph below.



5. Write the co-ordinates of the points A, B, C, D, E, F, G and H shown in the adjacent figure.



Solution:

The coordinate of point A is (2,2).
The coordinate of point B is (-3,0).
The coordinate of point C is (-2,-4).
The coordinate of point D is (3,-1).
The coordinate of point E is (-4,4).
The coordinate of point F is (0,-2).
The coordinate of point G is (2,-3).
The coordinate of point H is (0,3).

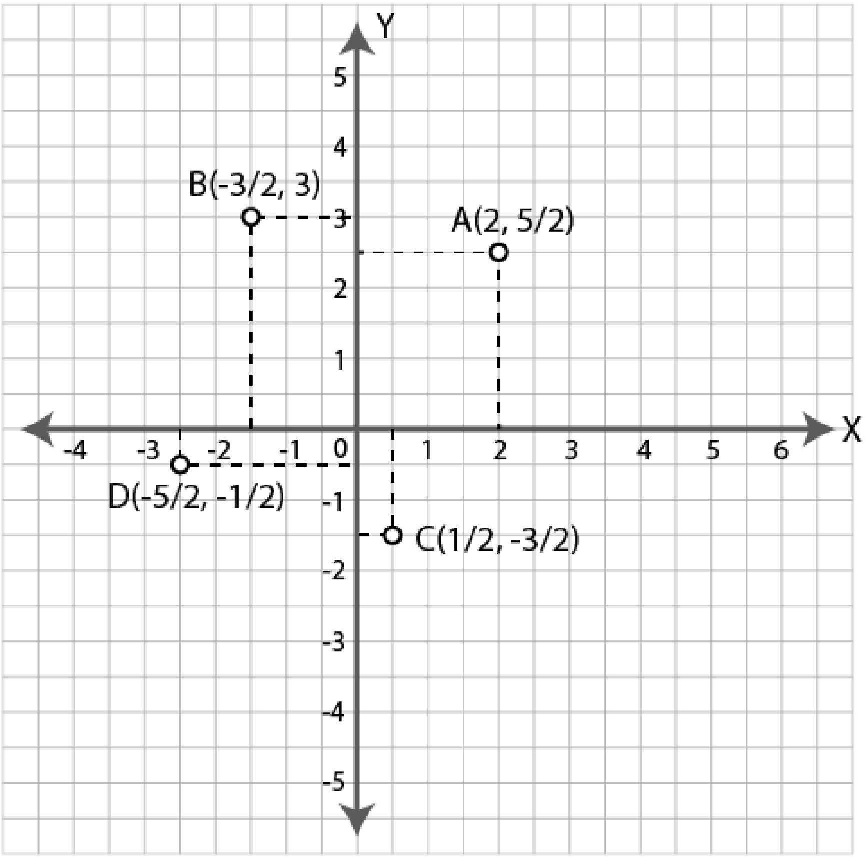
6. In which quadrants are the points A, B, C and D of problem 5 located ?

Solution:

For the point A (2,2), both x and y coordinate are positive. So it lies in the first quadrant.
For the point B(-3,0), y-coordinate is zero. So it lies on X-axis.
For the point C(-2,-4), both x and y coordinates are negative. So it lies in the third quadrant.
For the point D(3,-1), x coordinate is positive and y coordinate is negative. So it lies in the fourth quadrant.

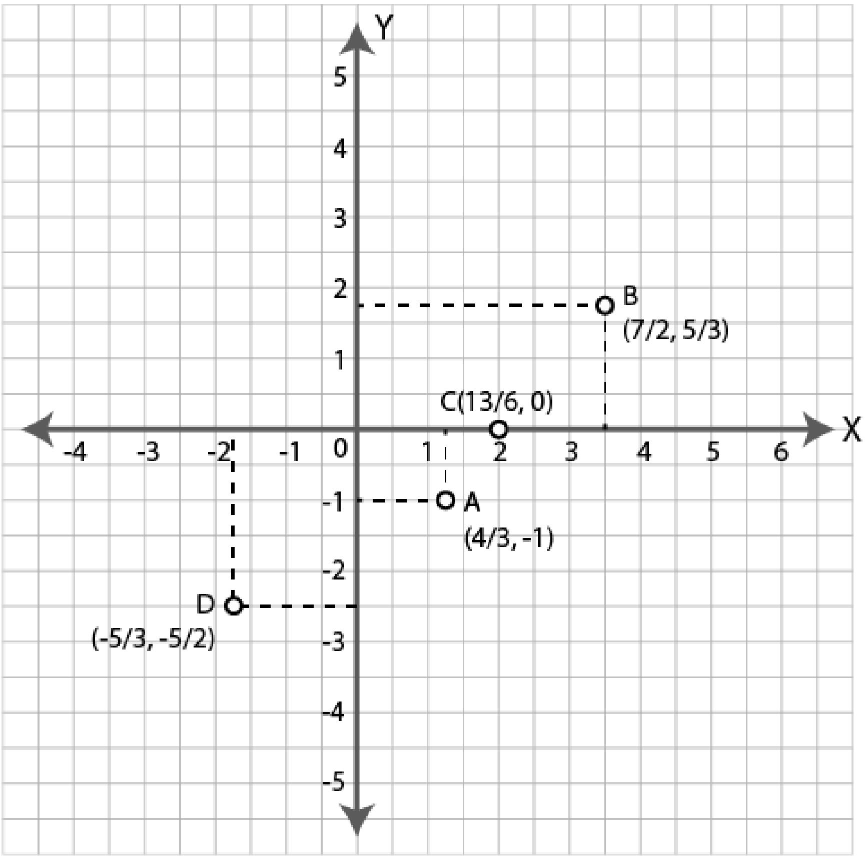
7. Plot the following points on the same graph paper :
A(2, 5/2), B(-3/2, 3), C(1/2, -3/2) and D(-5/2, -1/2).

Solution:
Given points are A(2, 5/2), B(-3/2, 3), C(1/2, -3/2) and D(-5/2, -1/2).
The points are plotted in the graph below.



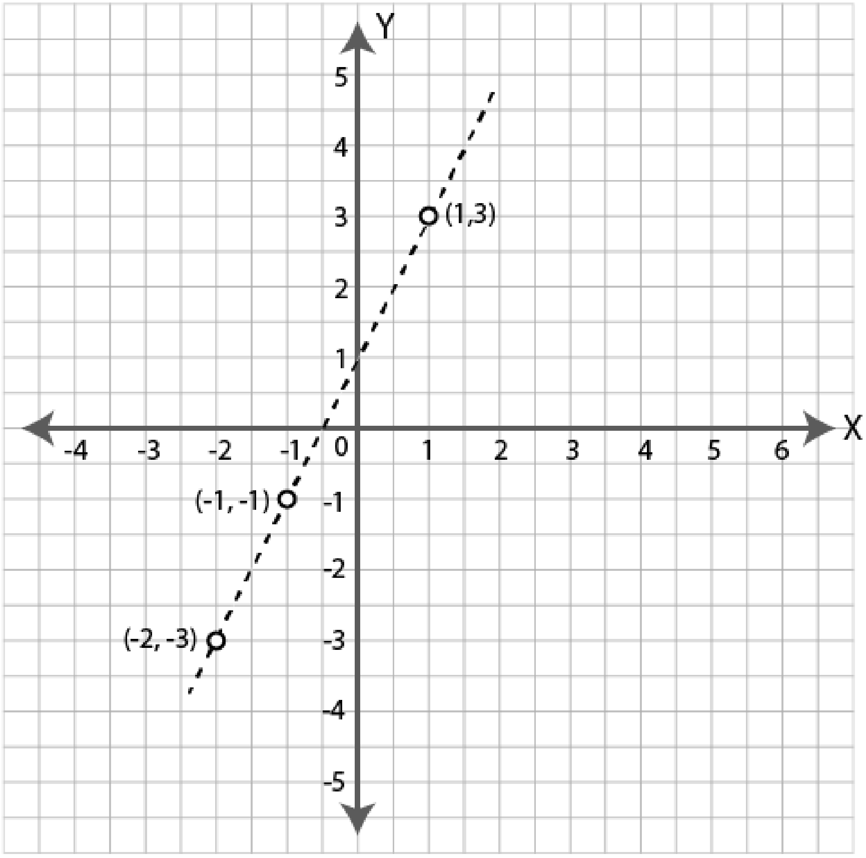
8. Plot the following points on the same graph paper.
A(4/3, -1), B(7/2, 5/3), C(13/6,0), D(-5/3,-5/2).

Solution:
Given points are A(4/3, -1), B(7/2, 5/3), C(13/6,0), D(-5/3,-5/2).
The points are plotted in the graph below.



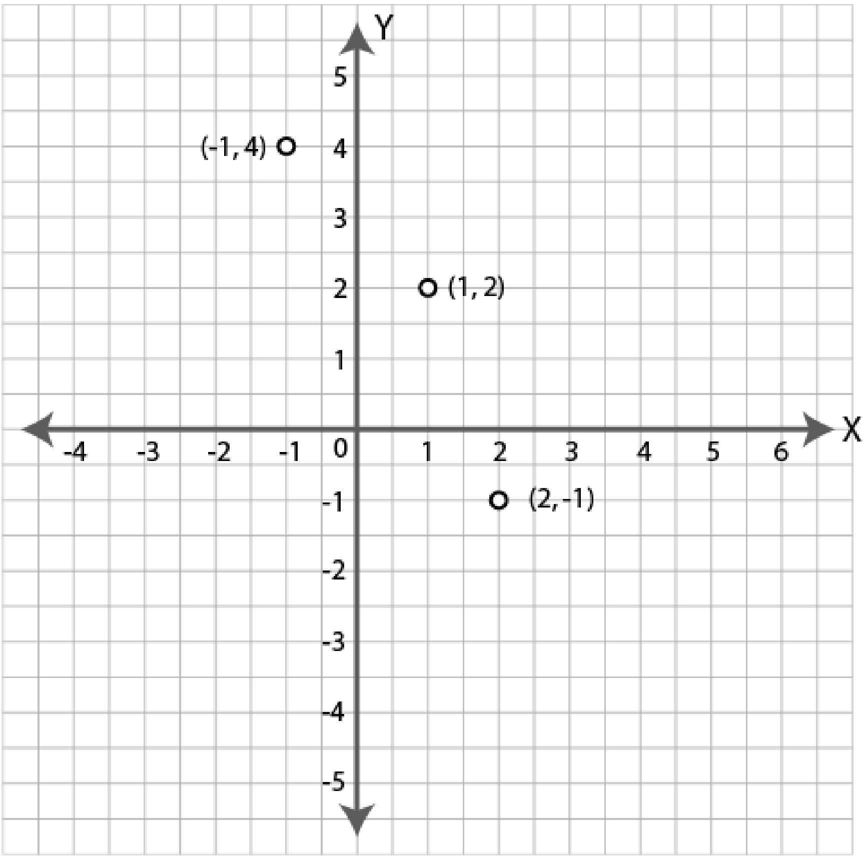
9.Plot the following points and check whether they are collinear or not:
(i) (1,3), (-1,-1) and (-2,-3)
(ii) (1,2), (2,-1) and (-1, 4)
(iii) (0,1), (2, -2) and (2/3,0).

Solution:
(i) (1,3), (-1,-1) and (-2,-3)



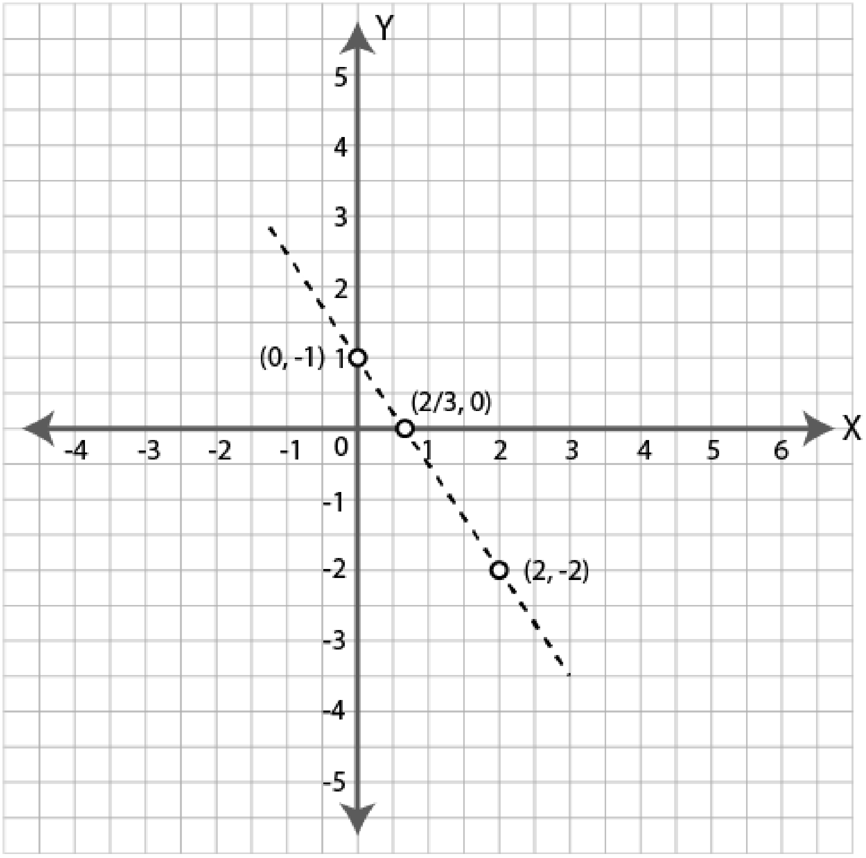
The given points lie on a line. So they are collinear.

(ii) (1,2), (2,-1) and (-1, 4)



The given points do not lie on a line. So they are not collinear.

(iii) (0,1), (2, -2) and (2/3,0).



The given points lie on a line. So they are collinear.

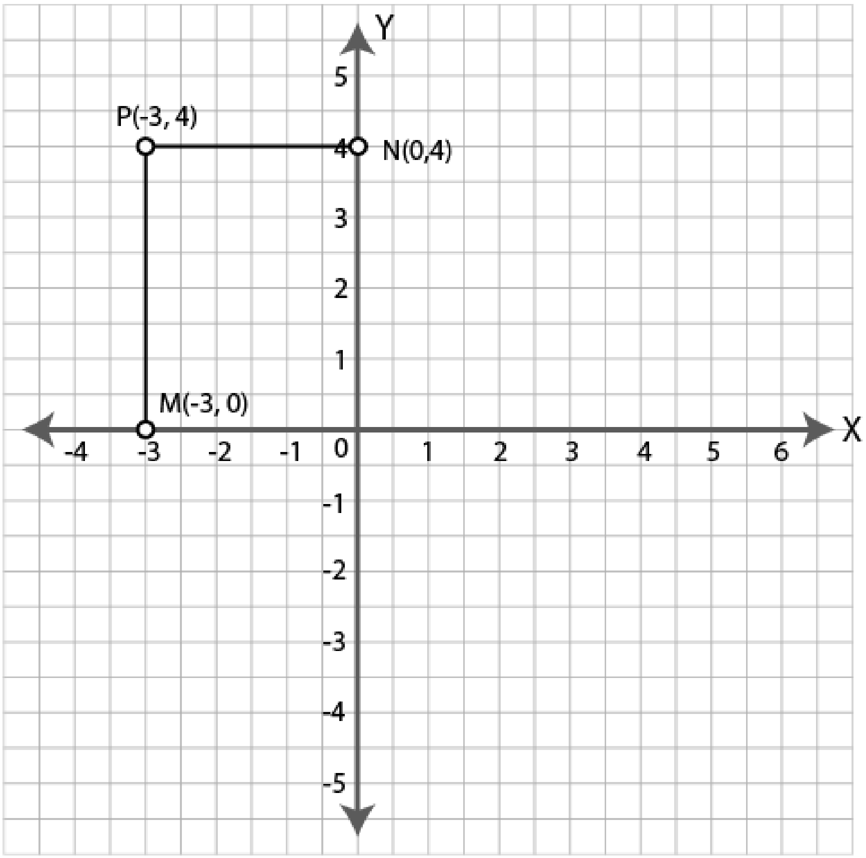
10. Plot the point P(-3, 4). Draw PM and PN perpendiculars to x-axis and y-axis respectively. State the co-ordinates of the points M and N.

Solution:

Given point is P(-3,4).

The point is plotted in the graph below.

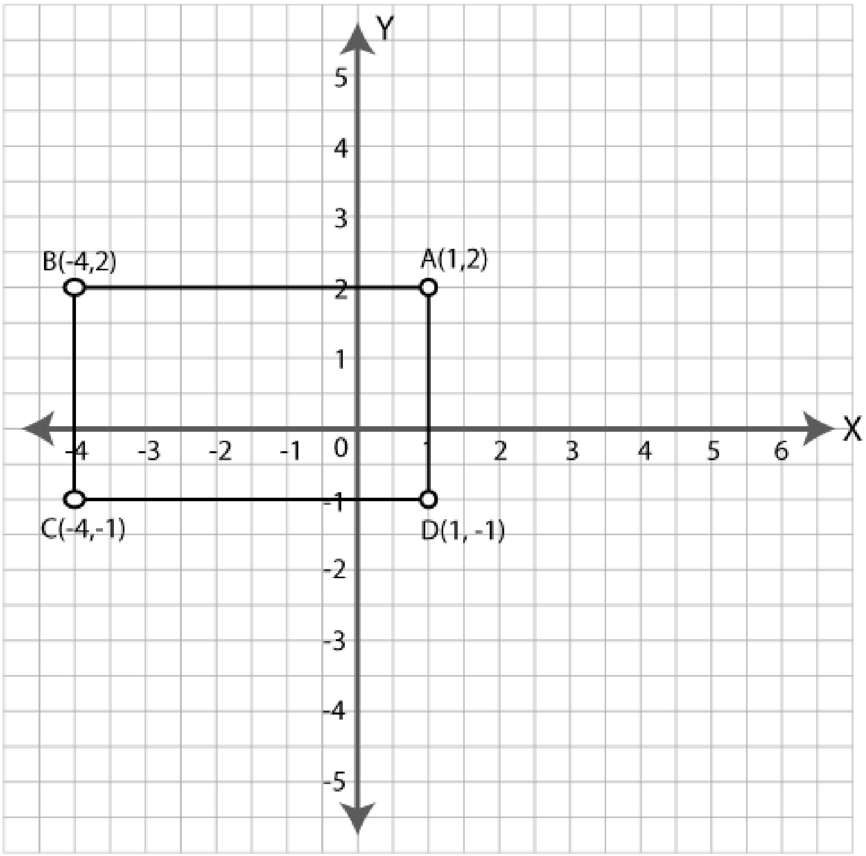
PM and PN is drawn perpendicular to x-axis and y-axis respectively.



Coordinates of point M are (-3,0) .
Coordinates of point N are (0,4).

11. Plot the points A (1,2), B (-4,2), C (-4, -1) and D (1, -1). What kind of quadrilateral is ABCD ? Also find the area of the quadrilateral ABCD.

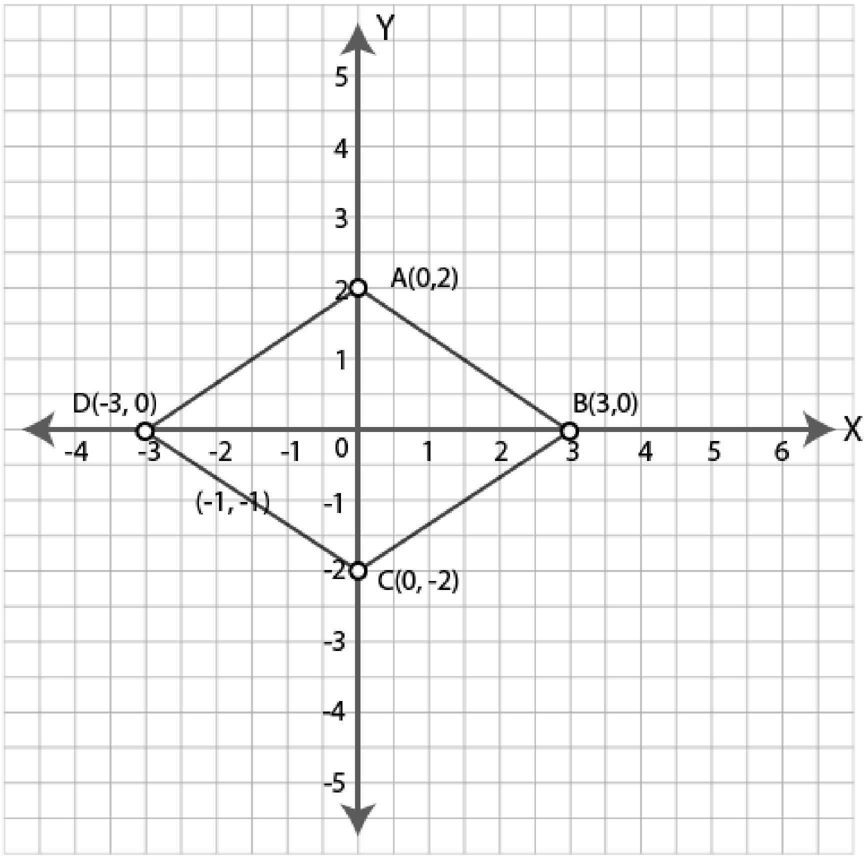
Solution:
Given points are A (1,2), B (-4,2), C (-4, -1) and D (1, -1).
The points are plotted in the graph below.



ABCD is a rectangle.
Area of rectangle ABCD = length ×breadth
= AB×AD
= (1-(-4))×(2-(-1))
= 5×3
= 15 sq. units.

12. Plot the points (0,2), (3,0), (0, -2) and (-3,0) on a graph paper. Join these points (in order). Name the figure so obtained and find the area of the figure obtained.

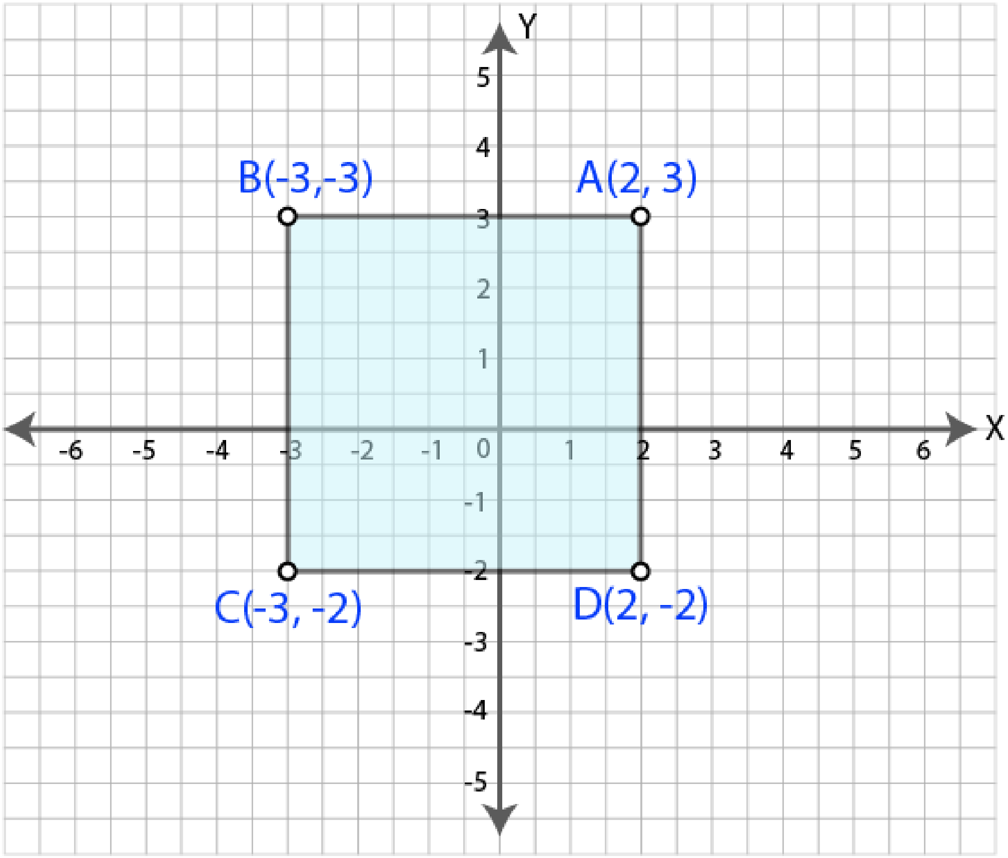
Solution:
Given points are (0,2), (3,0), (0,-2) and (-3,0).
The points are plotted in the graph below.



The quadrilateral obtained is a rhombus.
BD and AC are the diagonals of the rhombus.
Area of a rhombus = $\frac{1}{2} \times d_1 \times d_2$
Where d_1 and d_2 are the length of diagonals.
AC = 4 units [from graph]
BD = 6 units [from graph]
$\therefore$ Area of rhombus ABCD = $\frac{1}{2} \times BD \times AC$
$= \frac{1}{2} \times 6 \times 4$
= 12 sq. units.
Hence the area is 12 sq. units.

13. Three vertices of a square are A (2,3), B (-3, 3) and C (-3, -2). Plot these points on a graph paper and hence use it to find the co-ordinates of the fourth vertex. Also find the area of the square.

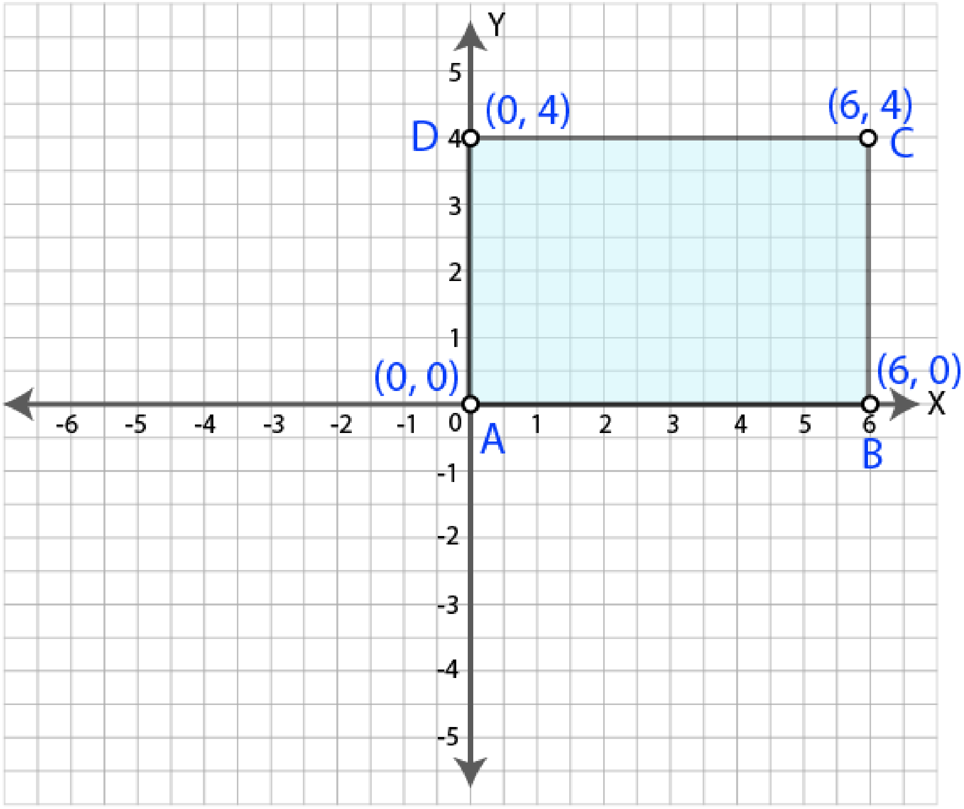
Solution:
Given points are A (2,3), B (-3, 3) and C (-3, -2).
The points are plotted on the graph below.



From the graph, the coordinates of point D are (2, -2).
Here AB = 5 units [from graph]
Area of the square = side ×side
Area of the square ABCD = AB×AB
= 5×5
= 25 sq. units.
Hence the area of the square is 25 sq. units.

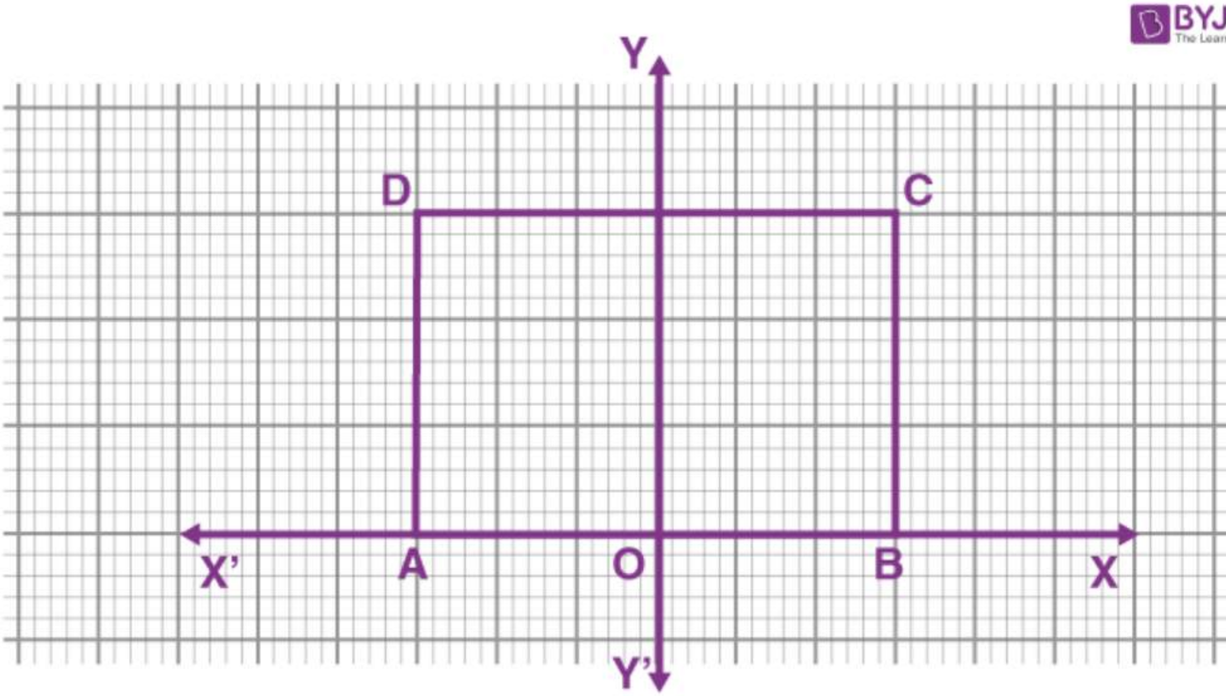
14. Write the co-ordinates of the vertices of a rectangle which is 6 units long and 4 units wide if the rectangle is in the first quadrant, its longer side lies on the x-axis and one vertex is at the origin.

Solution:
The rectangle which is 6 units long and 4 units wide is shown in the graph.
Rectangle is in the first quadrant.
Longer side lies on x -axis and one vertex is at origin.



Coordinates of the rectangle are A (0,0), B (6,0), C (6,4) and D (0,4).

15. In the adjoining figure, ABCD is a rectangle with length 6 units and breadth 3 units. If O is the mid-point of AB, find the coordinates of A, B, C and D.

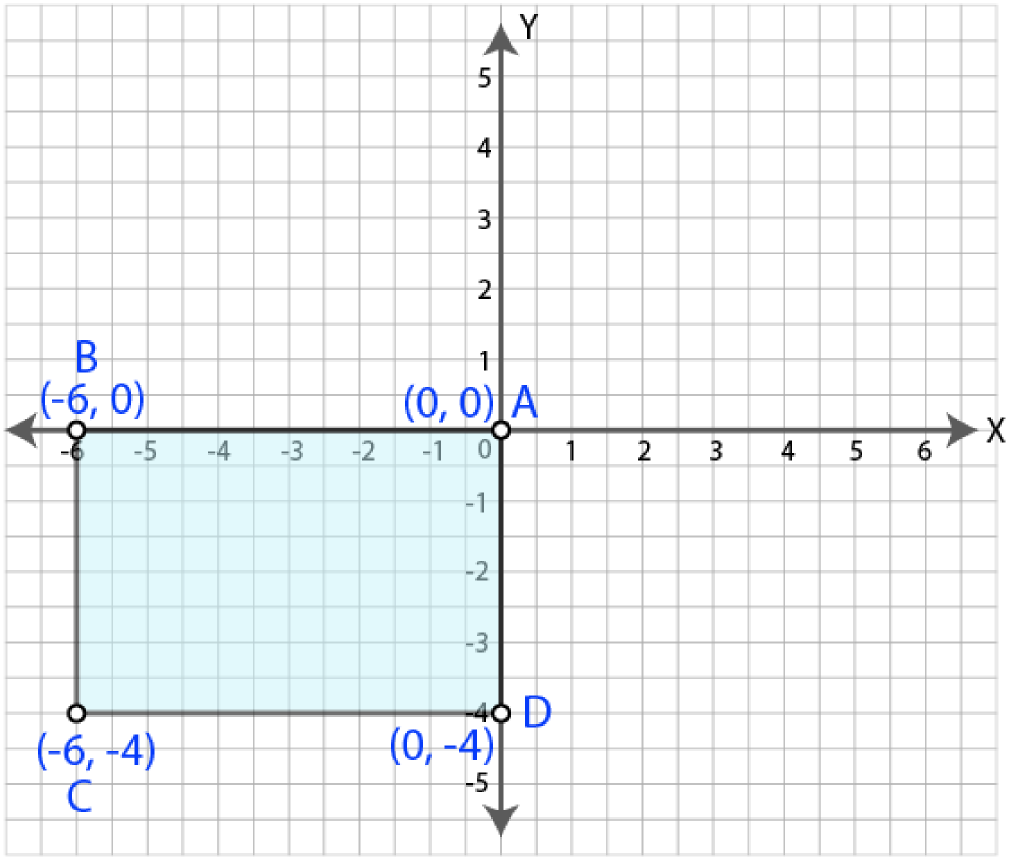


Solution:

The rectangle which is 6 units long and 4 units wide is shown in the graph.

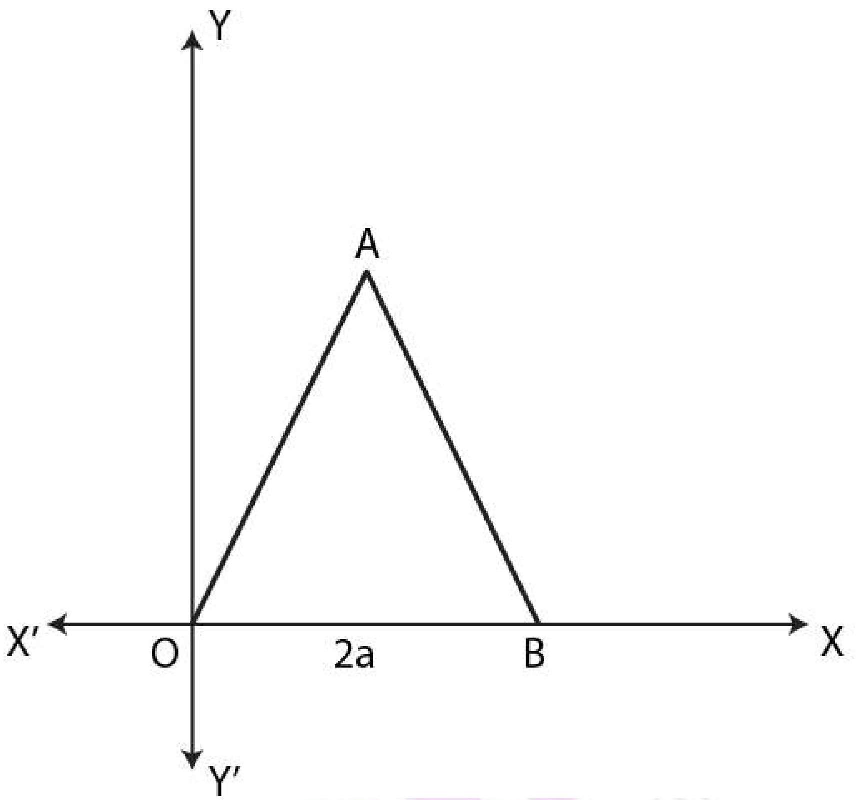
Rectangle is in the third quadrant.

Longer side lies on x -axis and one vertex is at origin.

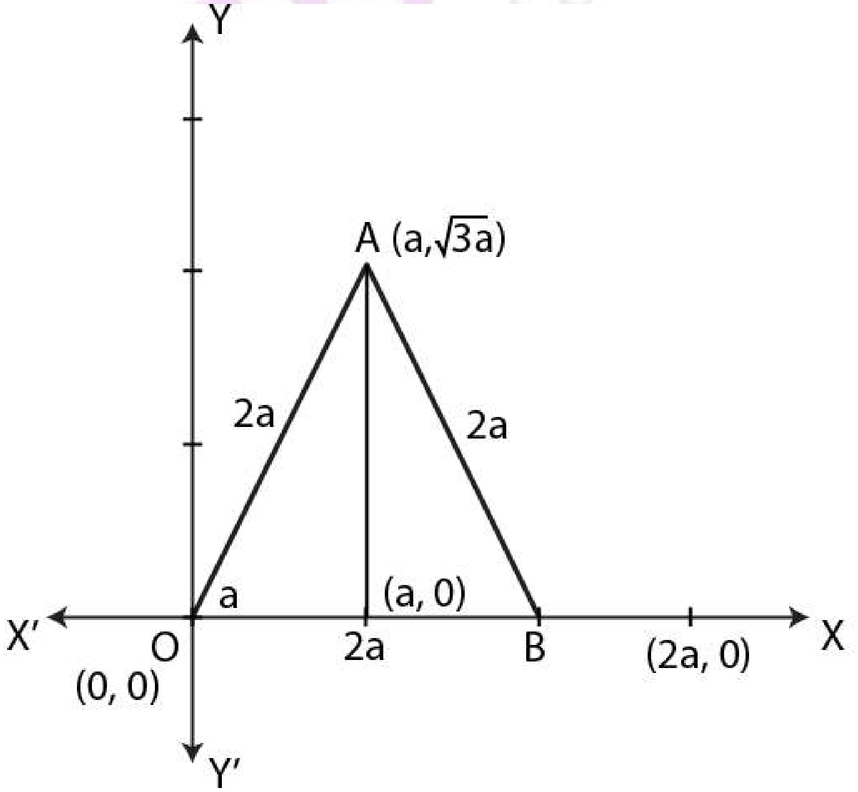


Coordinates of the rectangle are A(0,0), B(-6,0), C(-6,-4) and D(0,-4).

16. The adjoining figure shows an equilateral triangle OAB with each side = 2a units. Find the coordinates of the vertices.

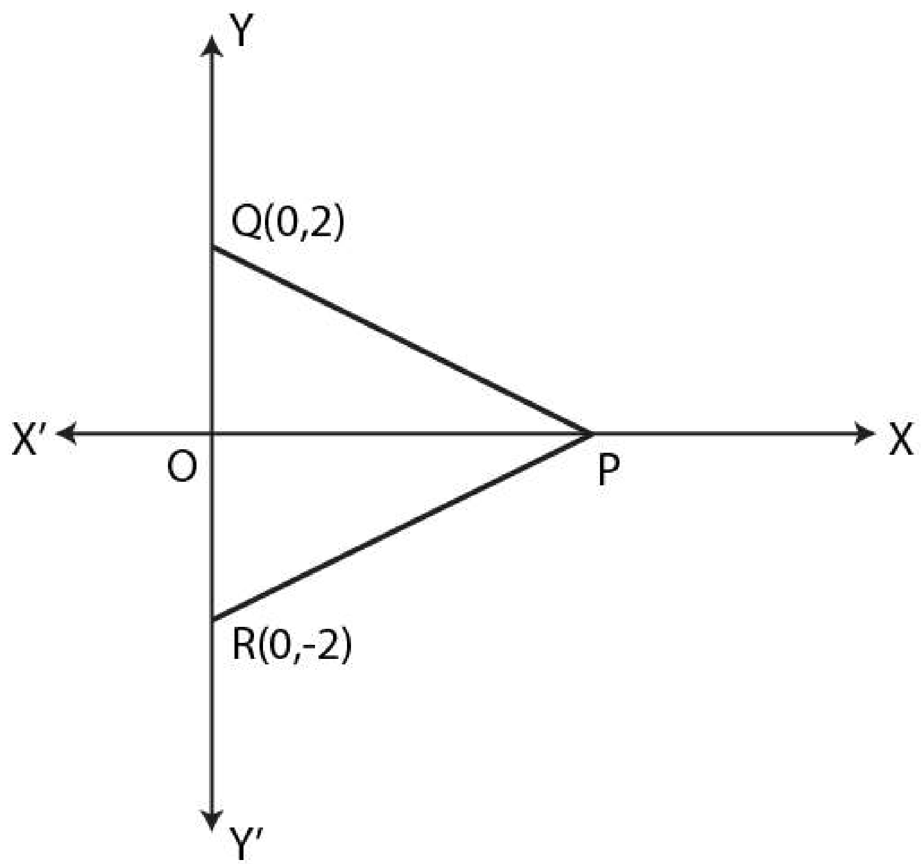


Solution:
Given equilateral triangle OAB.
OA = OB = AB = 2a units.



Draw AD ⊥ OB.
$AD = \sqrt{(AO^2-DO^2)}$
$= \sqrt{((2a)^2-a^2)}$
$= \sqrt{(4a^2-a^2)}$
$= \sqrt{(3a^2)}$
$= \sqrt{3}\ a$
Co-ordinates of O are (0,0).
Co-ordinates of A are $(a, \sqrt{3}\ a)$
Co-ordinates of B are (2a,0).

17. In the given figure, △PQR is equilateral. If the coordinates of the points Q and R are (0, 2) and (0, -2) respectively, find the coordinates of the point P.



Solution:
Given △PQR is an equilateral triangle in which Q(0,2) and R(0,-2).
Let (x,0) be the coordinates of P. [∵P lies on x axis. So y-coordinate is zero.]
∴ PQ = PR = QR = 2+2 = 4
OQ = 2 [from figure]
In △POQ,
$PQ^2 = OP^2+OQ^2$ [Pythagoras theorem]
$4^2 = OP^2+2^2$

$\therefore OP^2 = 4^2 - 2^2$
$\therefore OP^2 = 16 - 4$
$\therefore OP^2 = 12$
$OP = \sqrt{(4\times3)} = 2\sqrt{3}$
Hence the coordinates of P are $(2\sqrt{3}, 0)$.

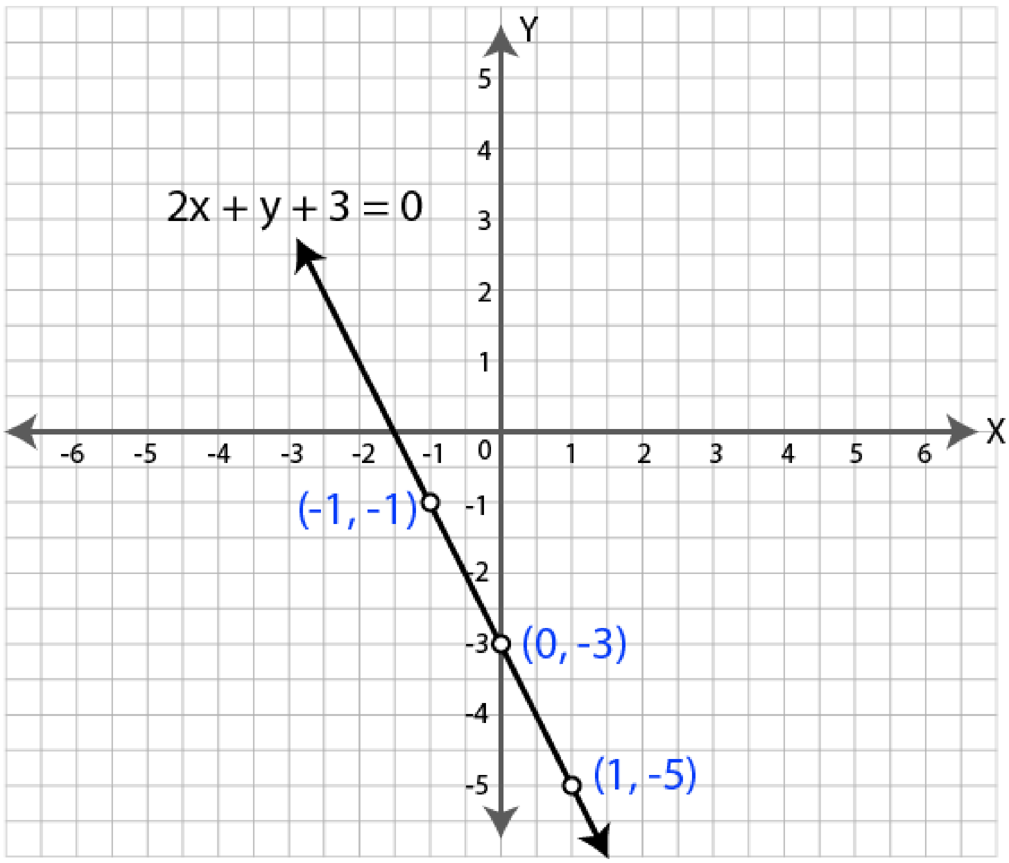
EXERCISE 19.2

1. Draw the graphs of the following linear equations:
(i) 2x +y+ 3 = 0
(ii) x- 5y- 4 = 0.

Solution:
(i)2x+y+3 = 0
y = -2x-3
Substitute some values for x and find y.
When x = -1 ,
y = -2×-1-3 = 2-3 = -1
when x = 0,
y = -2×0-3 = 0-3 = -3
when x = 1,
y = -2×1-3 = -2-3 = -5

x	-1	0	1
y	-1	-3	-5

Plot the graph using the values (-1,-1), (0,-3),and (1,-5) as shown below.

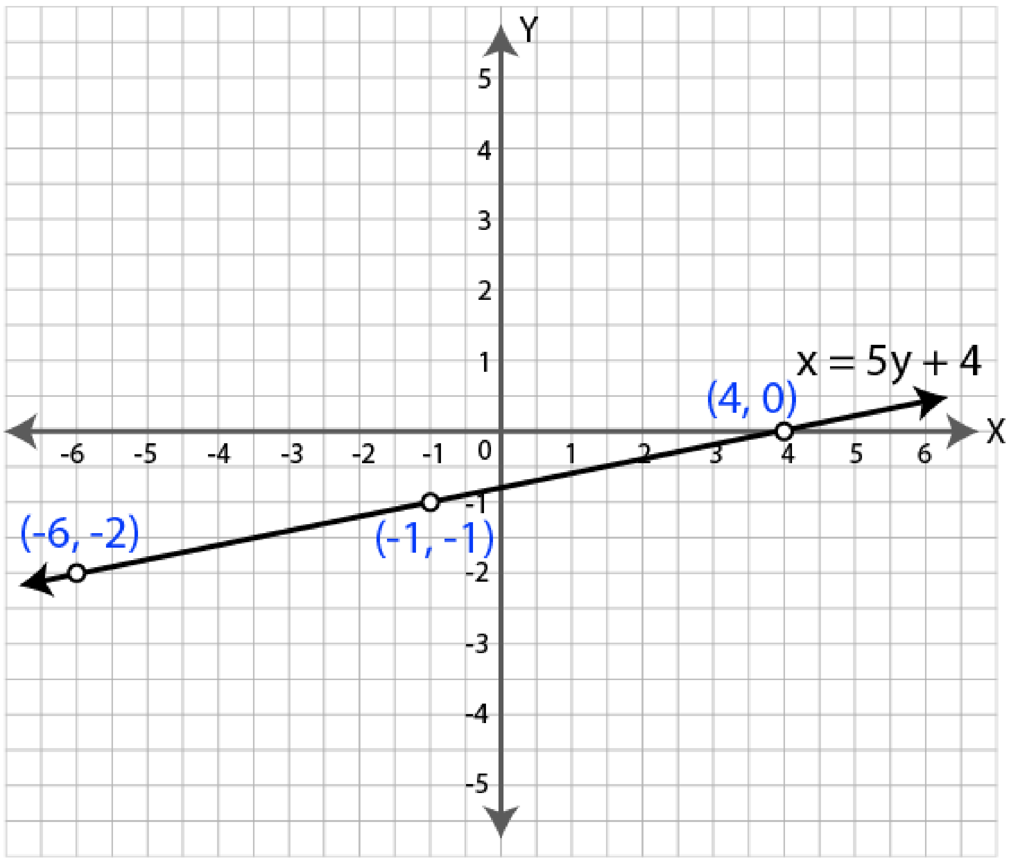


(ii)x-5y-4 = 0

$\Rightarrow x = 5y+4$
When y = -2 ,
x = 5×-2+4 = -10+4 = -6
When y = -1 ,
x = 5×-1+4 = -5+4 = -1
When y = 0,
x = 5×0+4 = 0+4 = 4

x	-6	-1	4
y	-2	-1	0

Plot the graph using the values (-6,-2), (-1,-1),and (4,0) as shown below.

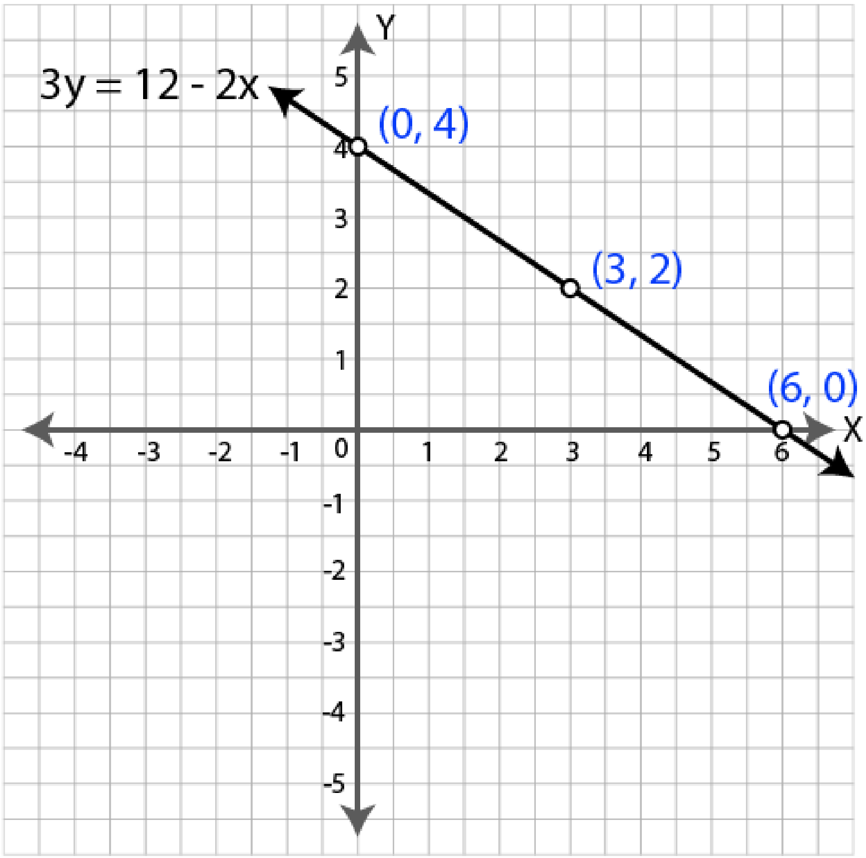


2. Draw the graph of 3y = 12-2x. Take 2cm = 1 unit on both axes.

Solution:
3y = 12-2x
y = (12-2x)/3
when x = 0,
y = (12-2×0)/3 = 12/3 = 4
when x = 3,
y = (12-2×3)/3 = 6/3 = 2
when x = 6,
y = (12-2×6)/3 = 0

x	0	3	6
y	4	2	0

Plot the graph using the values (0,4), (3,2) and (6,0) as shown below.



3. Draw the graph of 5x+6y-30 = 0 and use it to find the area of the triangle formed by the line and the co-ordinate axes.

Solution:

$5x+6y-30 = 0$

$5x = 30-6y$

$x = (30-6y)/5$

when $y = 0$,

$x = (30-6\times0)/5 = 30/5 = 6$

when $y = 5$,

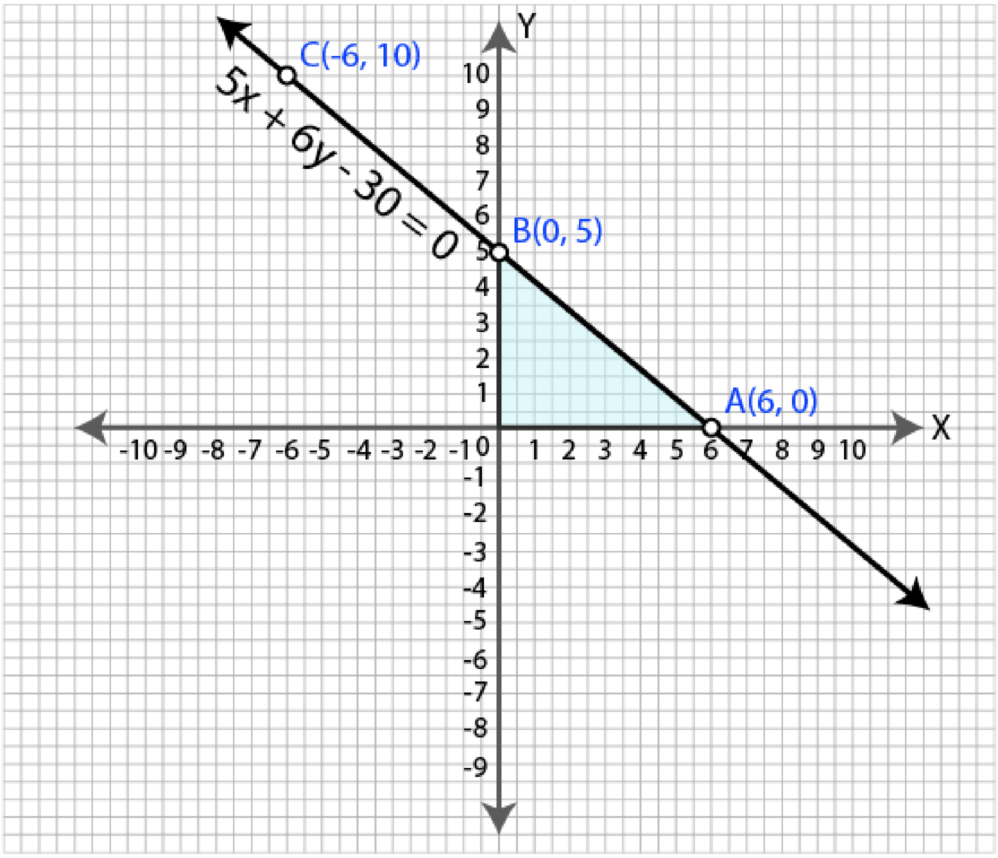
$x = (30-6\times5)/5 = 0$

when $y = 10$,

$x = (30-6\times10)/5 = -30/5 = -6$

x	6	0	-6
y	0	5	10

Plot the graph using the values (6,0), (0,5) and (-6,10) as shown below.



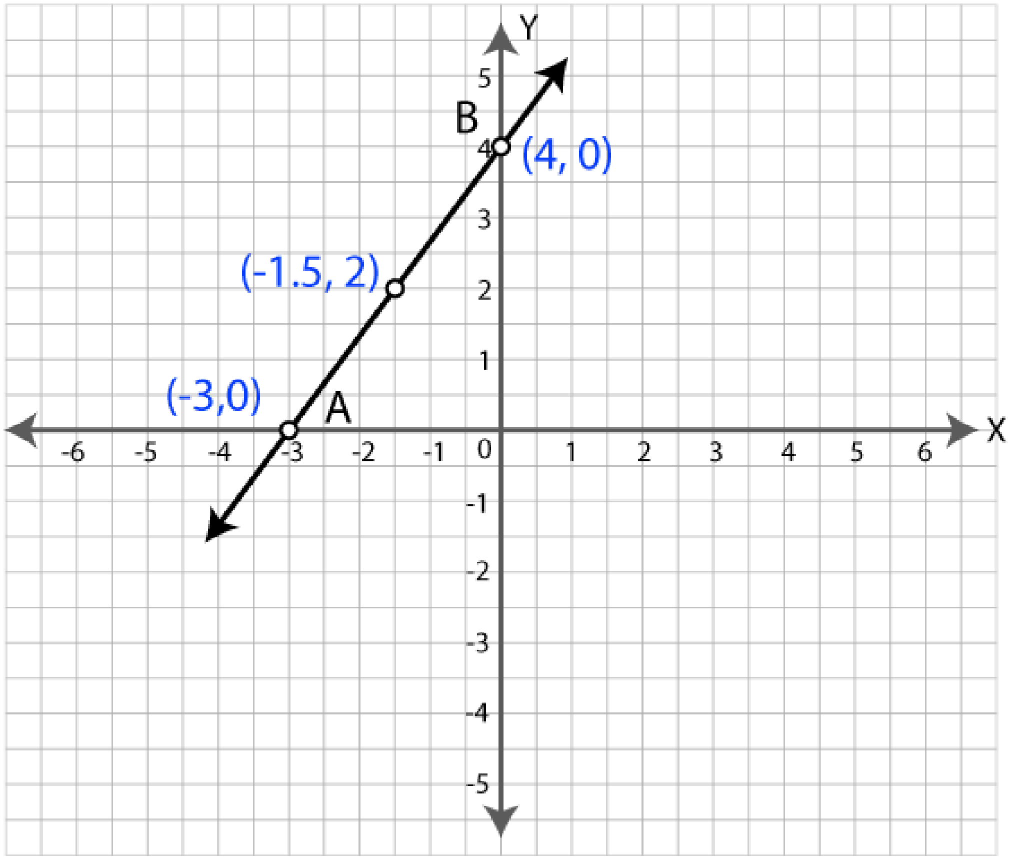
Area of the triangle formed by line and coordinate axes = ½ OA×OB
= ½ ×6×5
= 30/2
= 15 sq. units.
Hence area of the triangle is 15 sq. units.

4. Draw the graph of 4x-3y+12 = 0 and use it to find the area of the triangle formed by the line and the co-ordinate axes. Take 2 cm = 1 unit on both axes.

Solution:
4x-3y+12 = 0
4x = 3y-12
x = (3y-12)/4
when y = 0,
x = (3×0-12)/4 = -12/4 = -3
when y = 2,
x = (3×2-12)/4 = -6/4 = 1.5
when y = 4,
x = (3×4-12)/4 = 0

x	-3	-1.5	0
y	0	2	4

Plot the graph using the values (-3,0), (-1.5,2),and (0,4) as shown below.



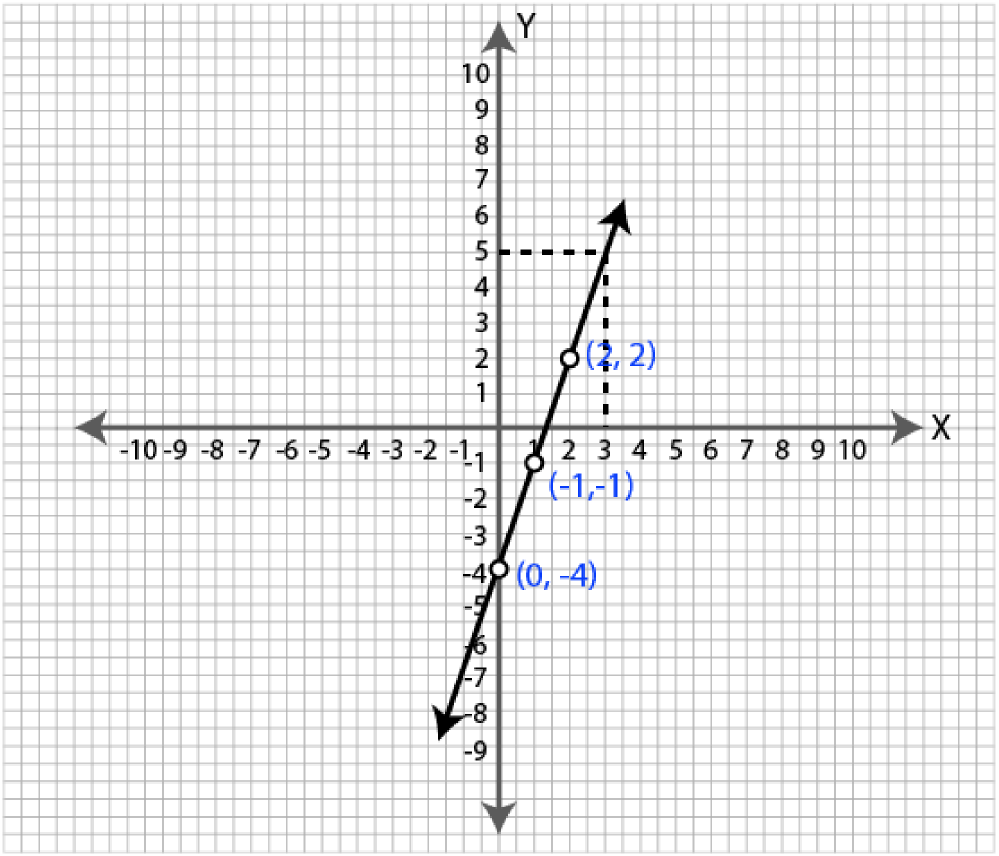
Area of the triangle formed by line and coordinate axes = ½ |OA| × |OB|
= ½ ×3×4
= 12/2
= 6 sq. units.
Hence area of the triangle is 6 sq. units.

5. Draw the graph of the equation y = 3x – 4. Find graphically.
(i) the value of y when x = -1
(ii) the value of x when y = 5.

Solution:
$y = 3x-4$
when $x = 0$,
$y = 3\times0-4 = 0-4 = -4$
when $x = 1$,
$y = 3\times1-4 = 3-4 = -1$
when $x = 2$,
$y = 3\times2-4 = 6-4 = 2$

x	0	1	2
y	-4	-1	2

Plot the graph using the values (0, -4), (1, -1) and (2,2) as shown below.

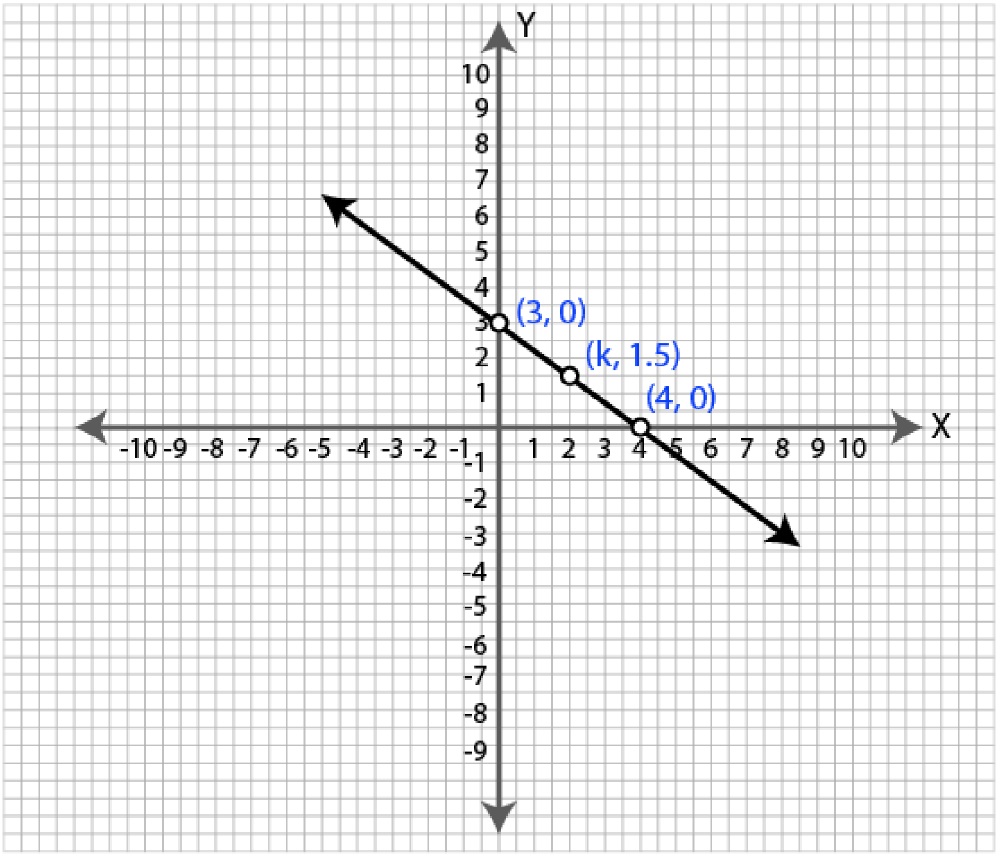


(i) x = -1:
Draw a line parallel to Y axis from x = -1. It meets the graph at y = -7.
So when x = -1, the value of y is -7.

(ii) y = 5
Draw a line parallel to X axis from y = 5. It meets the graph at x = 3.
So when y = 5, the value of x is 3.

6. The graph of a linear equation in x and y passes through (4, 0) and (0, 3). Find the value of k if the graph passes through (k, 1.5).

Solution:
Plot the points (4,0) and (0,3) on a graph.
Join them.

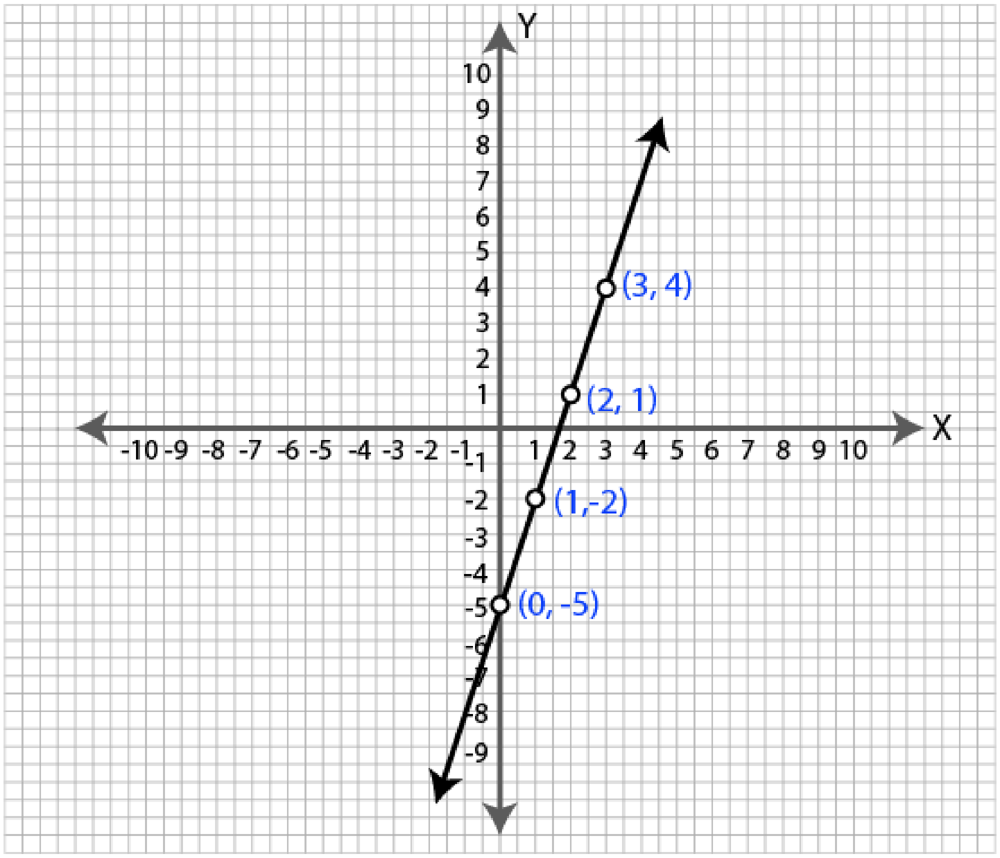


Mark A(k,1.5).
From the graph it is clear that the value of k is 2.

7. Use the table given alongside to draw the graph of a straight line. Find, graphically the values of a and b.

x	1	2	3	a
y	-2	b	4	-5

Solution:
Plot the points (1,-2), (2,b), (3,4) and (a,-5) on the graph.



From the graph, it is clear that value of a is 0 and b is 1.
Hence a = 0 and b = 1.

EXERCISE 19.3

1. Solve the following equations graphically: 3x-2y = 4, 5x-2y = 0

Solution:

3x-2y = 4 ..(i)

⇒2y = 3x-4

⇒y = (3x-4)/2

When x = 0,

y = (3×0-4)/2 = (0-4)/2 = -4/2 = -2

when x = 2,

y = (3×2-4)/2 = (6-4)/2 = 2/2 = 1

when x = 4,

y = (3×4-4)/2 = (12-4)/2 = 8/2 = 4

x	0	2	4
y	-2	1	4

Plot the above points on graph. Join them.

5x-2y = 0 …(ii)

⇒2y = 5x

⇒y = 5x/2

When x = 0,

y = 0

When x = 2,

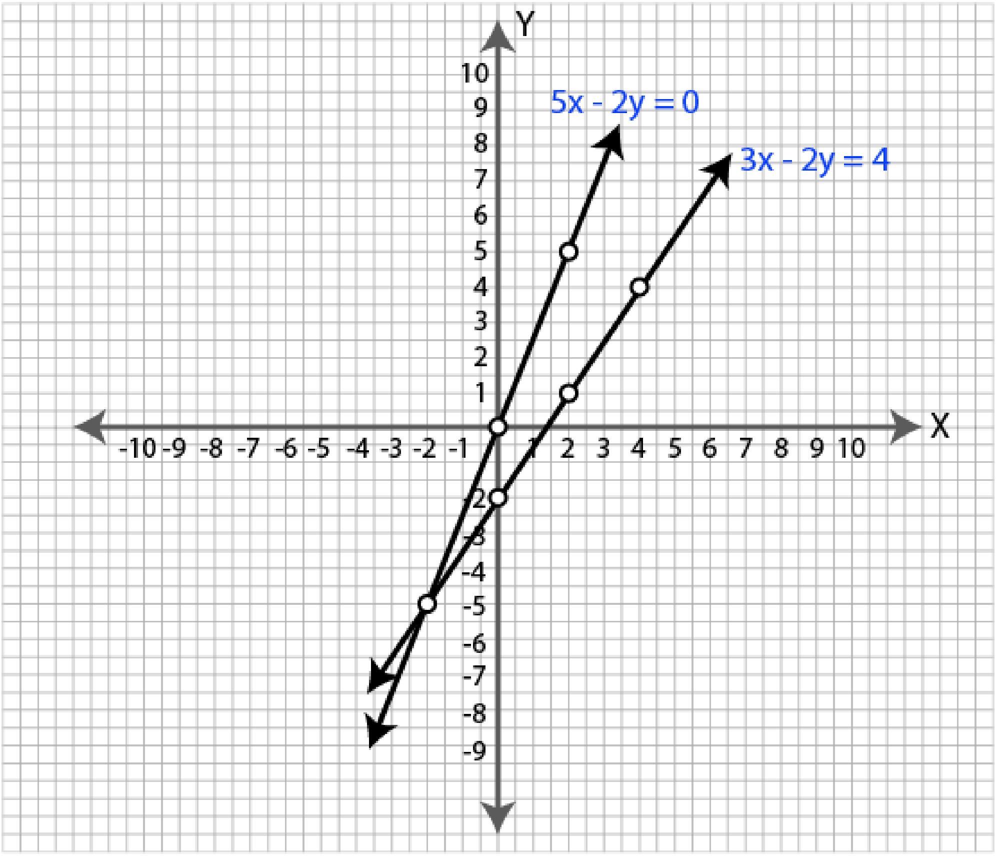
y = 5×2/2 = 5

When x = -2,

y = 5×-2/2 = -5

x	0	2	-2
y	0	5	-5

Plot the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (-2,-5).
So the solution of the given equations are x = -2 and y = -5.

2. Solve the following pair of equations graphically. Plot at least 3 points for each straight line 2x -7y = 6, 5x -8y = -4.

Solution:
2x-7y = 6 ….(i)
2x = 7y+6
x = (7y+6)/2
when y = 0
x = (7×0+6)/2 = 6/2 = 3
when y = -1
x = (7×-1+6)/2 = -1/2 = -0.5
when y = -2
x = (7×-2+6)/2 = -8/2 = -4

x	3	-0.5	-4
y	0	-1	-2

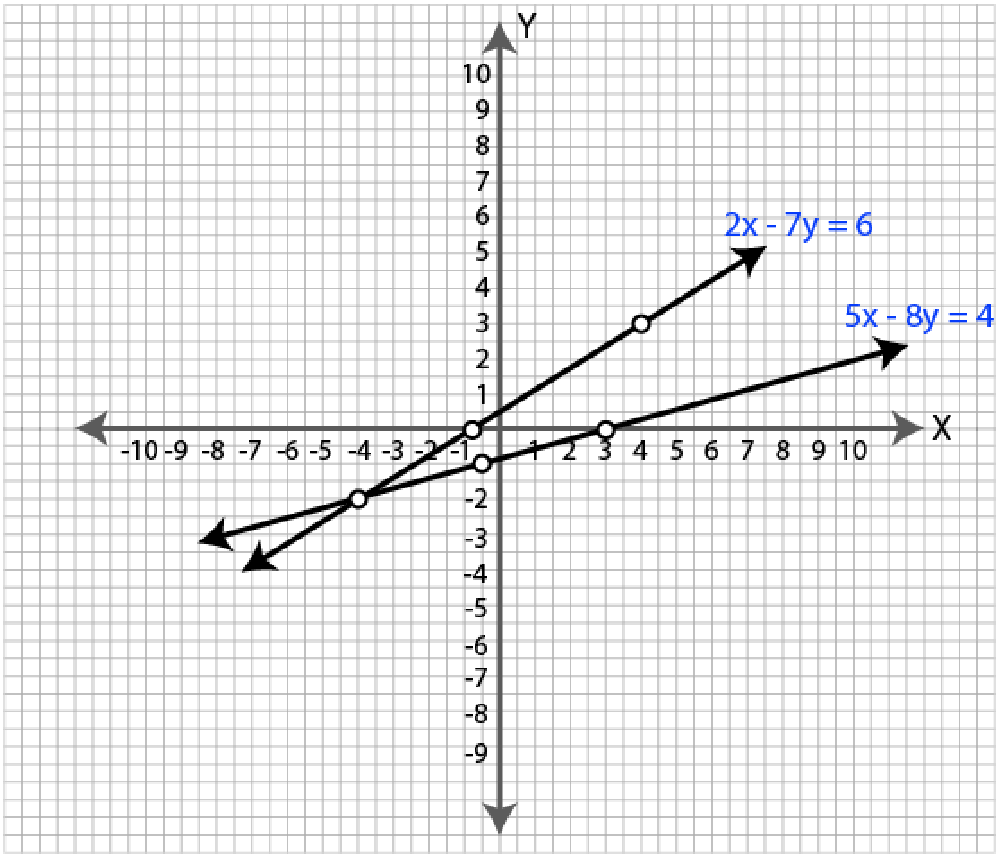
Mark the above points on graph. Join them.

5x-8y = -4 …(ii)
5x = 8y-4
x = (8y-4)/5

when y = 0
x = (8×0-4)/5 = -4/5 = 0.8
when y = 3
x = (8×3-4)/5 = (24-4)/5 = 20/5 = 4
when y = -2
x = (8×-2-4)/5 = (-16-4)/5 = -20/5 = -4

x	0.8	4	-4
y	0	3	-2

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (-4,-2).
So the solution of the given equations are x = -4 and y = -2.

3. Using the same axes of co-ordinates and the same unit, solve graphically.
x+y = 0, 3x – 2y = 10

Solution:
x+y = 0 ..(i)
⇒y = -x
When x = -3,
y = 3
When x = -2,

y = 2
When x = -1,
y = 1

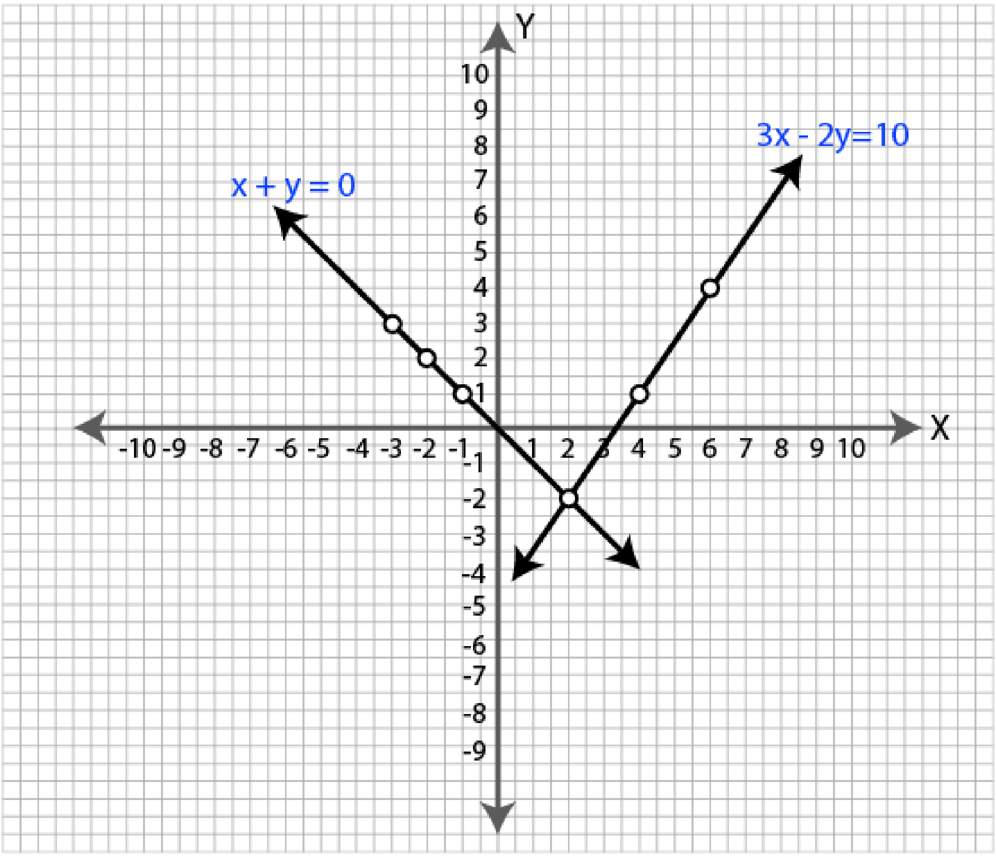
x	-3	-2	-1
y	3	2	1

Mark the above points on graph. Join them.

3x-2y = 10 ..(ii)
3x= 2y+10
⇒x = (2y+10)/3
When y = 1
x = (2×1+10)/3 = 12/3 = 4
When y = -2
x = (2×-2+10)/3 = 6/3 = 2
When y = 4
x = (2×4+10)/3 = 18/3 = 6

x	4	2	6
y	1	-2	4

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (2,-2).

So the solution of the given equations are x = 2 and y = -2.

4. Take 1 cm to represent 1 unit on each axis to draw the graphs of the equations 4x- 5y = -4 and 3x = 2y – 3 on the same graph sheet (same axes). Use your graph to find the solution of the above simultaneous equations.

Solution:

4x-5y = -4 ..(i)
4x = 5y-4
x = (5y-4)/4
When y = 0
x = (5×0-4)/4 = -4/4 = -1
When y = 2
x = (5×2-4)/4 = 6/4 = 1.5
When y = -2
x = (5×-2-4)/4 = -14/4 = -3.5

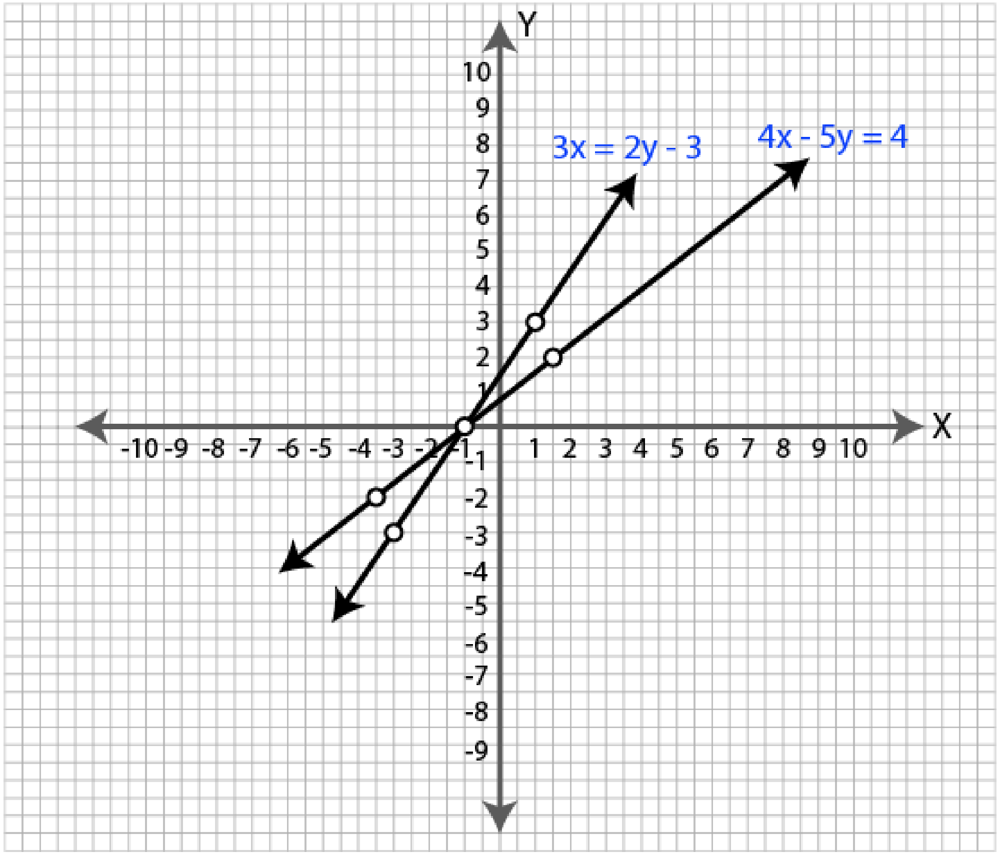
x	-3.5	-1	1.5
y	-2	0	2

Mark the above points on graph. Join them.

3x = 2y-3 …(ii)
⇒x = (2y-3)/3
When y = 0,
x = (2×0-3)/3 = -3/3 = -1
When y = 3,
x = (2×3-3)/3 = 3/3 = 1
When y = -3,
x = (2×-3-3)/3 = -9/3 = -3

x	-1	1	-3
y	0	3	-3

Mark the above points on graph. Join them



It is clear from the graph that the two lines intersect at (-1,0).
So the solution of the given equations are x = -1 and y = 0.

5. Solve the following simultaneous equations graphically, x + 3y = 8, 3x = 2 + 2y.

Solution:

$x+3y = 8$ …(i)

$\Rightarrow 3y = 8-x$

$\Rightarrow y = (8-x)/3$

When x = 8,

$y = (8-8)/3 = 0$

when x = 2,

$y = (8-2)/3 = 6/3 = 2$

when x = 5,

$y = (8-5)/3 = 3/3 = 1$

x	2	5	8
y	2	1	0

Mark the above points on graph. Join them.

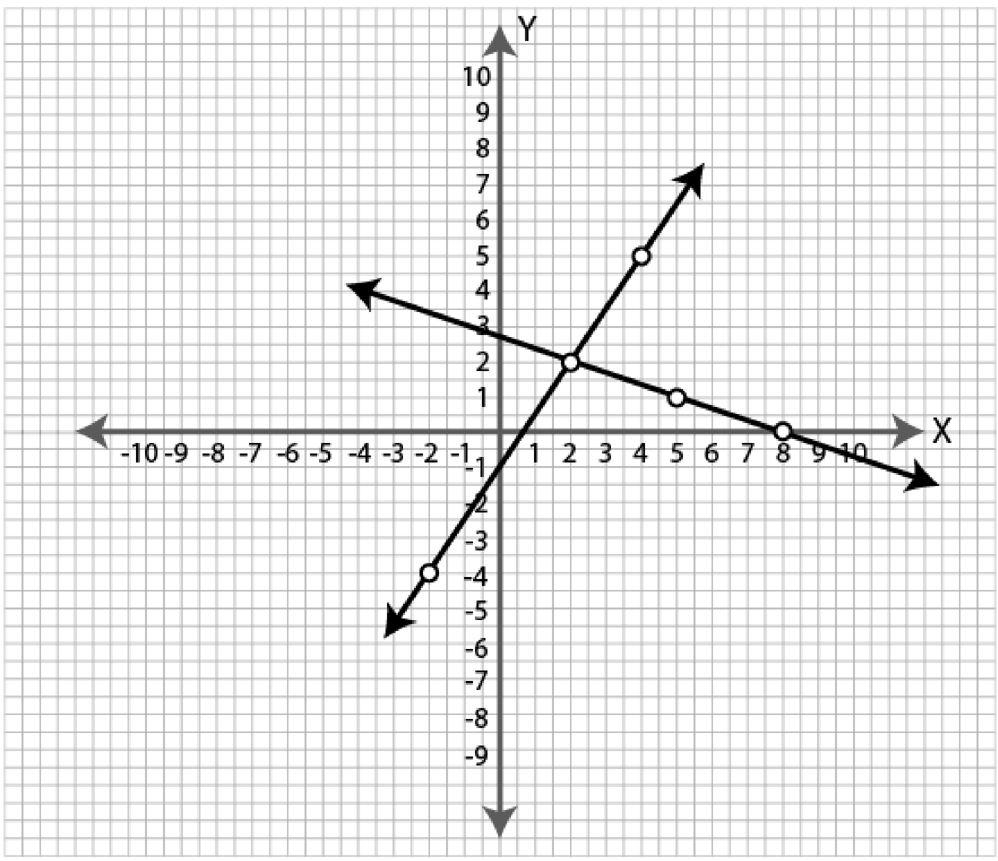
$3x = 2+2y$

$\Rightarrow 2y = 3x-2$

$\Rightarrow y = (3x-2)/2$
When x = 2
$y = (3\times2-2)/2 = 4/2 = 2$
When x = 4
$y = (3\times4-2)/2 = 10/2 = 5$
When x = -2
$y = (3\times-2-2)/2 = -8/2 = -4$

x	-2	2	4
y	-4	2	5

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (2,2).
So the solution of the given equations are x = 2 and y = 2.

6. Solve graphically the simultaneous equations 3y = 5 – x, 2x = y + 3 (Take 2cm = 1 unit on both axes).

Solution:
$3y = 5-x$
$\Rightarrow y = (5-x)/3$
When x = 5,
$y = (5-5)/3 = 0$
When x = 2,

y = (5-2)/3 = 3/3 = 1
When x = -1,
y = (5-(-1))/3 =6/3 = 2

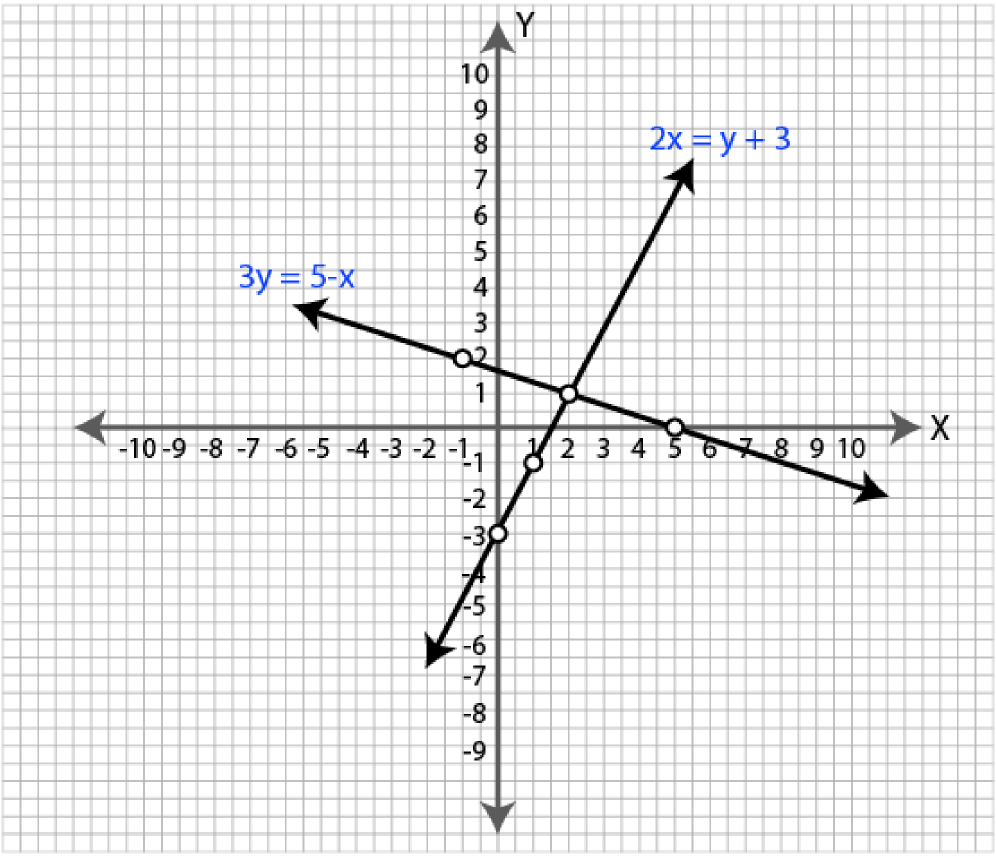
x	-1	2	5
y	2	1	0

Mark the above points on graph. Join them.

2x = y+3 …(ii)
⇒y = 2x-3
When x = 0,
y = (2×0-3) = 0-3 = -3
When x = 1,
y = (2×1-3) = 2-3 = -1
When x = 2,
y = (2×2-3) = 4-3 = 1

x	0	1	2
y	-3	-1	1

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (2,1).
So the solution of the given equations are x = 2 and y = 1.

7.Use graph paper for this question.
Take 2 cm = 1 unit on both axes.
(i) Draw the graphs of x +y + 3 = 0 and 3x-2y + 4 = 0. Plot only three points per line.
(ii) Write down the co-ordinates of the point of intersection of the lines.
(iii) Measure and record the distance of the point of intersection of the lines from the origin in cm.

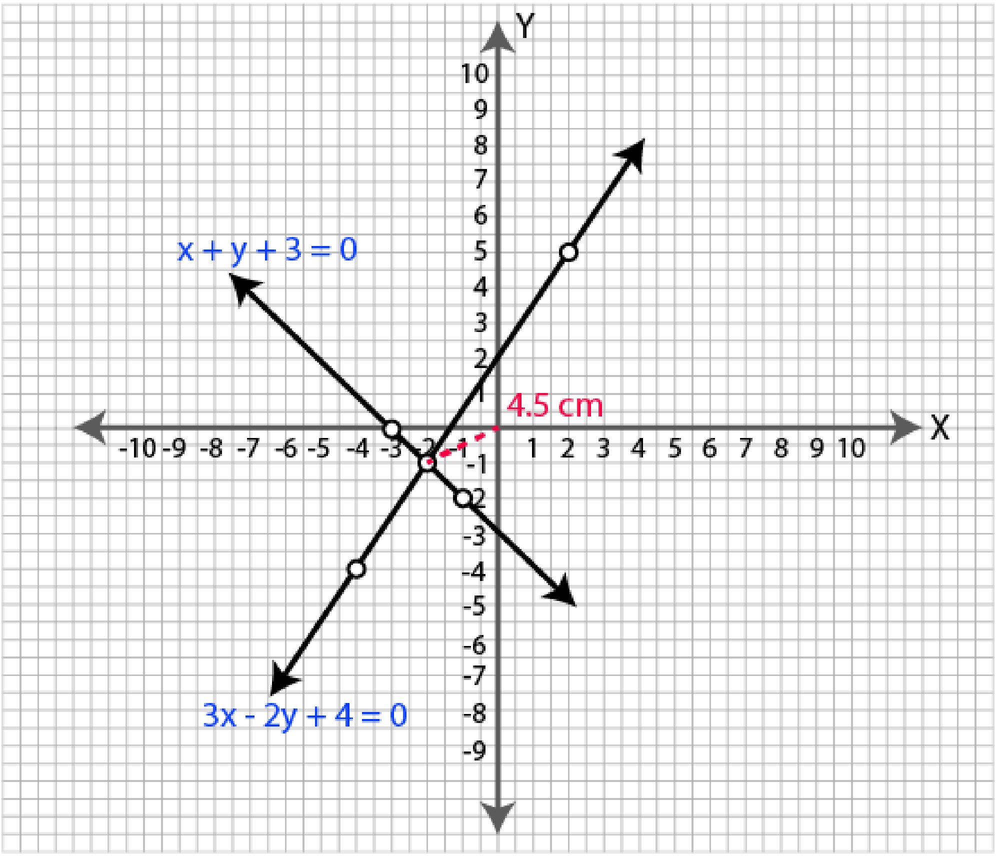
Solution:
(i) x+y+3 = 0 …(i)
⇒y = -x-3
When x = -3
y = 3-3 = 0
when x = -2
y = 2-3 = -1
when x = -1
y = 1-3 = -2

x	-1	-2	-3
y	-2	-1	0

Mark the above points on graph. Join them.
3x-2y+4 = 0 …(ii)
⇒2y = 3x+4
⇒y = (3x+4)/2
When x = -4
y = (3×-4+4)/2 = (-12+4)/2 = -8/2 = -4
When x = -2
y = (3×-2+4)/2 = (-6+4)/2 = -2/2 = -1
When x = 2
y = (3×2+4)/2 = (6+4)/2 = 10/2 = 5

x	-4	-2	2
y	-4	-1	5

Mark the above points on graph. Join them.



(ii)The two lines intersect at (-2,-1).
(iii)Measure the distance from origin to the point (-2,-1).
The distance of the point of intersection of the lines from the origin is 4.5 cm.

8. Solve the following simultaneous equations, graphically:
2x-3y + 2 = 4x+ 1 = 3x – y + 2

Solution:
Consider first equation.
2x-3y+2 = 4x+1
⇒3y = 2x-4x+2-1
⇒3y = -2x+1
⇒y = (-2x+1)/3
When x = -1,
y = (-2×-1+1)/3 = 3/3 = 1
When x = 2,
y = (-2×2+1)/3 = -3/3 = -1
When x = 0.5,
y = (-2×0.5+1)/3 = 0

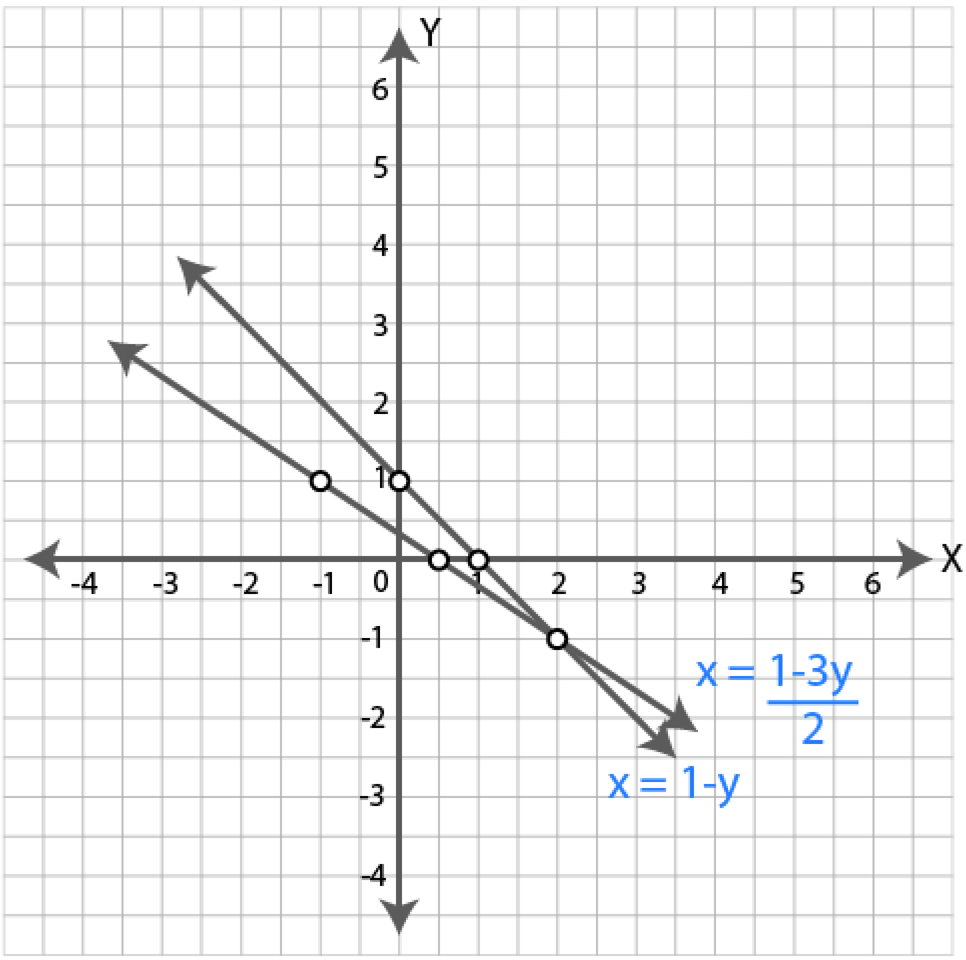
x	0.5	2	-1
y	0	-1	1

Mark the above points on graph. Join them.

Consider second equation.
4x+1 = 3x-y+2
⇒y = 3x-4x+2-1
⇒y = -x+1
When x = 0
y = 0+1 = 1
When x = 1
y = -1+1 = 0
When x = 2
y = -2+1 = -1

x	0	1	2
y	1	0	-1

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (2,-1).
So the solution of the given equations are x = 2 and y = -1.

9. Use graph paper for this question.

(i) Draw the graphs of 3x -y – 2 = 0 and 2x + y – 8 = 0. Take 1 cm = 1 unit on both axes and plot three points per line.
(ii) Write down the co-ordinates of the point of intersection and the area of the triangle formed by the lines and the x-axis

Solution:
(i)3x-y-2 = 0 …(i)
⇒y = 3x-2
When x = 0, y = 3×0-2 = 0-2 = -2
When x = 1, y = 3×1-2 = 3-2 = 1
When x = 2, y = 3×2-2 = 6-2 = 4

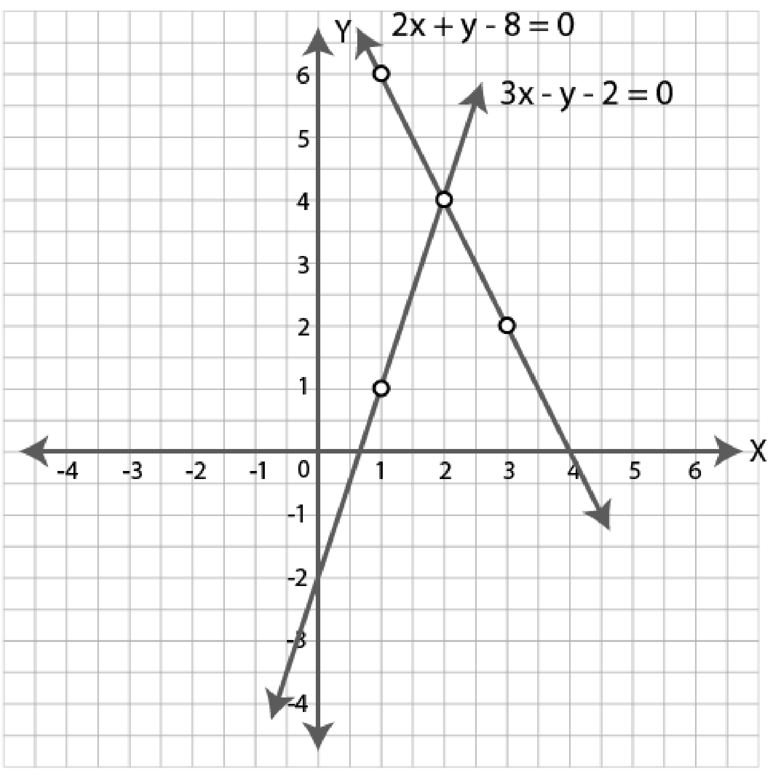
x	0	1	2
y	-2	1	4

Mark the above points on graph. Join them.

2x+y-8 = 0 ...(ii)
⇒y = -2x+8
When x = 1, y = -2×1+8 = -2+8 = 6
When x = 2, y = -2×2+8 = -4+8 = 4
When x = 3, y = -2×3+8 = -6+8 = 2

x	1	2	3
y	6	4	2

Mark the above points on graph. Join them.



(ii) The coordinates of the points of intersection are (2,4).
Area of the triangle formed = ½ ×base×height

= ½ ×3.4×4
= 6.8 sq. units
Hence area of the triangle is 6.8 sq. units.

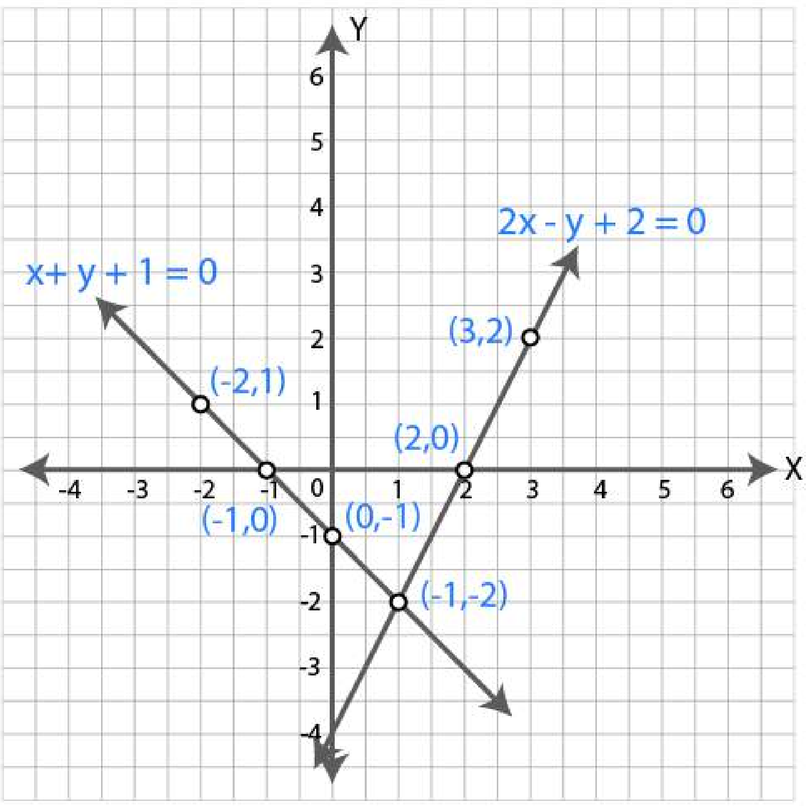
10. Solve the following system of linear equations graphically: 2x -y – 4 = 0, x + y + 1 = 0. Hence, find the area of the triangle formed by these lines and the y-axis.
Solution:
2x-y-4 = 0 …(i)
⇒y = 2x-4
When x = 1, y = 2×1-4 = 2-4 = -2
When x = 2, y = 2×2-4 = 4-4 = 0
When x = 3, y = 2×3-4 = 6-4 = 2

x	1	2	3
y	-2	0	2

Mark the above points on graph. Join them.
x+y+1 = 0 …(ii)
⇒y = -x-1
When x = 0, y = 0-1 = -1
When x = -2, y = 2-1 = 1
When x = -1, y = 1-1 = 0

x	-2	-1	0
y	1	0	-1

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (1,-2).
So the solution of the given equations are x = 1 and y = -2.
The area of the triangle formed by these lines and Y axis = ½ ×base×height
= ½ ×3×1
= 1.5 sq. units
Hence area of the triangle is 1.5 sq. units.

11. Solve graphically the following equations: x + 2y = 4, 3x – 2y = 4
Take 2 cm = 1 unit on each axis. Write down the area of the triangle formed by the lines and the x-axis.

Solution:
x+2y = 4 …(i)
⇒2y = 4-x
⇒y = (4-x)/2
When x = 0, y = (4-0)/2 = 4/2 = 2
When x = 2, y = (4-2)/2 = 2/2 = 1
When x = 4, y = (4-4)/2 = 0/2 = 0

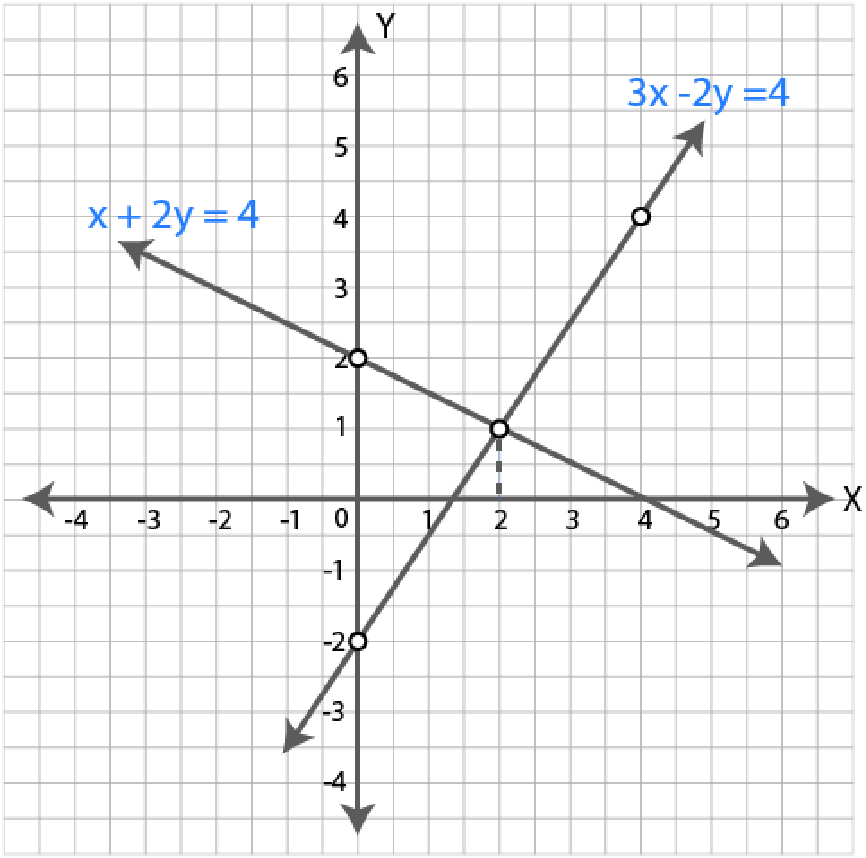
x	0	2	4
y	2	1	0

Mark the above points on graph. Join them.

3x-2y = 4 ..(ii)
⇒2y = 3x-4
⇒y = (3x-4)/2
When x = 0, y = (3×0-4)/2 = (0-4)/2 = -4/2 = -2
When x = 2, y = (3×2-4)/2 = (6-4)/2 = 2/2 = 1
When x = 4, y = (3×4-4)/2 = (12-4)/2 = 8/2 = 4

x	0	2	4
y	-2	1	4

Mark the above points on graph. Join them.



It is clear from the graph that the two lines intersect at (2,1).
So the solution of the given equations are x = 2 and y = 1.
The area of the triangle formed by these lines and X axis = ½ ×base×height
= ½ ×2.7×1
= 1.35 sq. units
Hence area of the triangle is 1.35 sq. units.

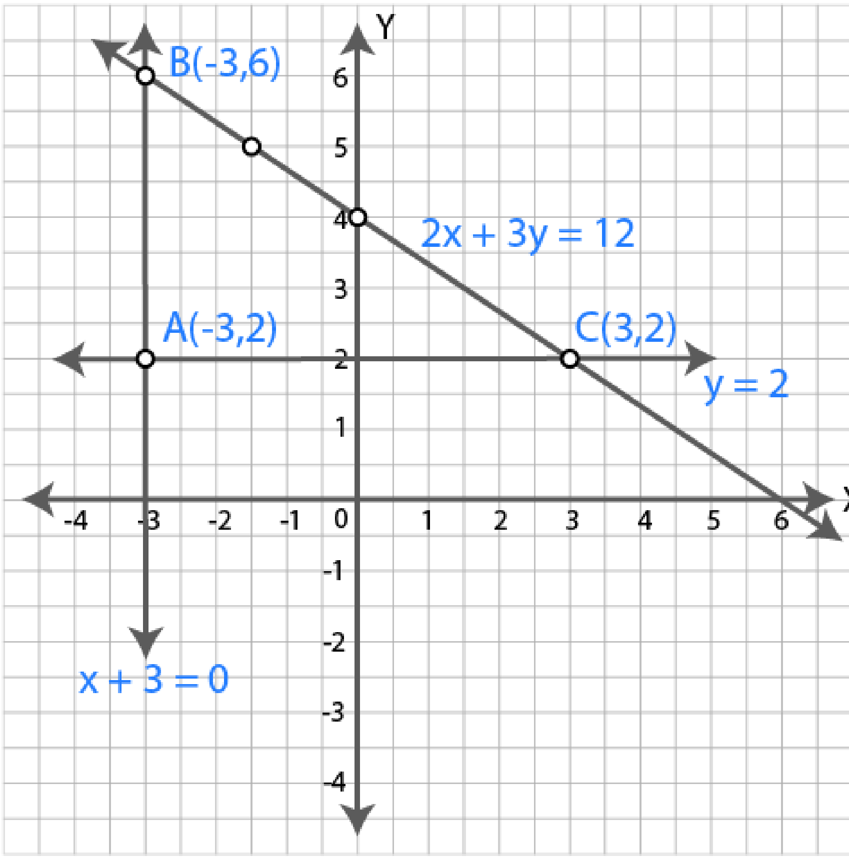
12. On graph paper, take 2 cm to represent one unit on both the axes, draw the lines : x + 3 = 0, y – 2 = 0, 2x + 3y = 12 .
Write down the co-ordinates of the vertices of the triangle formed by these lines.

Solution:
x+3 = 0 ..(i)
⇒ x = -3
The graph of x = -3 will be a line passing through x = -3 parallel to Y axis.
y-2 = 0 ..(ii)
⇒y = 2
The graph of y = 2 will be a line passing through y = 2 parallel to X axis.
2x+3y = 12 …(iii)
⇒3y = 12-2x
⇒y = (12-2x)/3
When x = 0, y = (12-2×0)/3 = 12/3 = 4

When x = 3, y = (12-2×3)/3 = (12-6)/3 = 6/3 = 2
When x = 6, y = (12-2×6)/3 = (12-12)/3 = 0

x	0	3	6
y	4	2	0

Mark the above points on graph. Join them.



From the graph, it is clear that the vertices of the triangle formed by the lines are A(-3,2), B(-3,6) and C(3,2).

13. Find graphically the co-ordinates of the vertices of the triangle formed by the lines y = 0, y = x and 2x + 3y= 10. Hence find the area of the triangle formed by these lines.

Solution:
y = 0 ..(i)
The graph of y = 0 is the X axis.
y = x ..(ii)
When x = 1, y = 1.
When x = 2, y = 2.
When x = 3, y = 3.

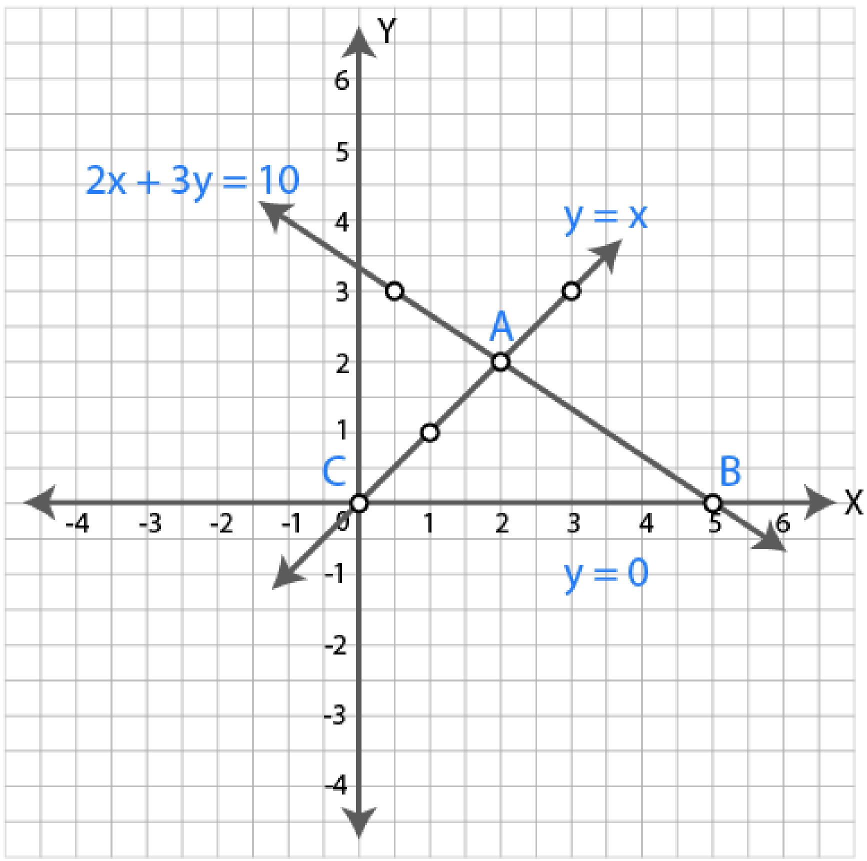
x	1	2	3
y	1	2	3

Mark the above points on graph. Join them.

$2x+3y = 10$ ..(iii)
$\Rightarrow 3y = 10-2x$
$\Rightarrow y = (10-2x)/3$
When $x = 0.5$, $y = (10-2\times0.5)/3 = (10-1)/3 = 9/3 = 3$
When $x = 2$, $y = (10-2\times2)/3 = (10-4)/3 = 6/3 = 2$
When $x = 5$, $y = (10-2\times5)/3 = (10-10)/3 = 0$

x	0.5	2	5
y	3	2	0

Mark the above points on graph. Join them.



From the graph, it is clear that the vertices of the triangle formed by the lines are A(2,2), B(5,0) and C(0,0).
Area of triangle formed by these lines = ½ ×base×height
= ½ ×5×2
= 5 sq. units
Hence area of the triangle is 5 sq. units.

EXERCISE 19.4

1. Find the distance between the following pairs of points:
(i) (2, 3), (4, 1)
(ii) (0, 0), (36, 15)
(iii) (a, b), (-a, -b)

Solution:
(i)Let $P(x_1, y_1)$ and $Q(x_2 , y_2)$ be the given points
Co-ordinates of P = (2,3)
Co-ordinates of Q = (4,1)
Here $x_1 = 2$, $y_1 = 3$, $x_2 = 4$, $y_2 = 1$
By distance formula $d(P,Q) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(P,Q) = \sqrt{[(4-2)^2+(1-3)^2]}$
$= \sqrt{[(2)^2+(-2)^2]}$
$= \sqrt{(4+4)}$
$= \sqrt{8}$
$= \sqrt{(4\times2)}$
$= 2\sqrt{2}$
Hence the distance between P and Q is $2\sqrt{2}$ units.

(ii) Let $P(x_1, y_1)$ and $Q(x_2 , y_2)$ be the given points
Co-ordinates of P = (0,0)
Co-ordinates of Q = (36,15)
Here $x_1 = 0$, $y_1 = 0$, $x_2 = 36$, $y_2 = 15$
By distance formula $d(P,Q) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(P,Q) = \sqrt{[(36-0)^2+(15-0)^2]}$
$= \sqrt{[(36)^2+(15)^2]}$
$= \sqrt{(1296+225)}$
$= \sqrt{1521}$
$= 39$
Hence the distance between P and Q is 39 units.

(iii) Let $P(x_1, y_1)$ and $Q(x_2 , y_2)$ be the given points
Co-ordinates of P = (a,b)
Co-ordinates of Q = (-a,-b)
Here $x_1 = a$, $y_1 = b$, $x_2 = -a$, $y_2 = -b$
By distance formula $d(P,Q) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(P,Q) = \sqrt{[(-a-a)^2+(-b-b)^2]}$
$= \sqrt{[(-2a)^2+(-2b)^2]}$
$= \sqrt{(4a^2+4b^2)}$
$= \sqrt{4(a^2+b^2)}$
$= 2\sqrt{(a^2+b^2)}$
Hence the distance between P and Q is $2\sqrt{(a^2+b^2)}$ units.

2. A is a point on y-axis whose ordinate is 4 and B is a point on x-axis whose abscissa is -3. Find the length of the line segment AB.

Solution:

Given A is a point on Y axis and ordinate is 4.
So the x-coordinate is 0.
$\therefore$ coordinates of A are (0,4)
Given B is a point on X axis and abscissa is -3.
So the y-coordinate is 0.
$\therefore$ coordinates of B are (-3,0)
By distance formula , Length of AB, d(AB) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore d(AB) = \sqrt{[(-3-0)^2+(0-4)^2]}$
$= \sqrt{(-3^2+-4^2)}$
$= \sqrt{(9+16)}$
$= \sqrt{25}$
$= 5$
Hence the length of line segment AB is 5 units.

3. Find the value of a, if the distance between the points A (-3, -14) and B (a, -5) is 9 units.

Solution:
Given distance between A(-3,-14) and B(a,-5) is 9 units.
By distance formula , Length of AB, d(AB) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore 9 = \sqrt{[(a-(-3))^2+(-5-(-14))^2]}$
$9 = \sqrt{[(a+3)^2+(-5+14)^2)]}$
$9 = \sqrt{[(a+3)^2+9^2]}$
$9 = \sqrt{[(a^2+6a+9+81)]}$
$9 = \sqrt{[(a^2+6a+90)]}$
Squaring both sides,
$81 = a^2+6a+90$
$a^2+6a+90-81 = 0$
$a^2+6a+9 = 0$
$(a+3)(a+3) = 0$
$\Rightarrow a+3 = 0$
$\Rightarrow a = -3$
Hence the value of a is -3.

4. (i) Find points on the x-axis which are at a distance of 5 units from the point (5, -4).
(ii) Find points on the y-axis which are at a distance of 10 units from the point (8, 8) ?
(iii) Find points (or points) which are at a distance of $\sqrt{10}$ from the point (4, 3) given that the ordinate of the point or points is twice the abscissa.

Solution:
(i)Given the point is on x axis. So y-coordinate is 0.
$\therefore$ Let the points on X-axis be A(x,0) which is at a distance of 5 units from B(5,-4).
By distance formula, distance between AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore 5 = \sqrt{[(5-x)^2+(-4-0)^2]}$
$\therefore 5 = \sqrt{[(5-x)^2+-4^2]}$
$\therefore 5 = \sqrt{[(25+x^2-10x+16)]}$
$\therefore 5 = \sqrt{[(x^2-10x+41)]}$
Squaring both sides
$25 = x^2-10x+41$
$\therefore\ x^2-10x+41-25 =0$

$\therefore\ x^2-10x+16=0$
$\therefore (x-2)(x-8) = 0$
$x-2 = 0$ or $x-8 = 0$
$x = 2$ or $x = 8$
Hence the points are (2,0) and (8,0).

(ii) Given the point is on Y axis. So x-coordinate is 0.
Let the points on Y-axis be A(0,y) which is at a distance of 10 units from B(8,8).
By distance formula , distance between AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore 10= \sqrt{[(8-0)^2+(8-y)^2]}$
$\therefore 10= \sqrt{[(8)^2+(8-y^2]}$
$\therefore 10 = \sqrt{[(64+64+y^2-16y)]}$
$\therefore 10 = \sqrt{[(y^2-16y+128)]}$
Squaring both sides
$100 = y^2-16y+128$
$\therefore\ y^2-16y+128-100 =0$
$\therefore\ y^2-16y+28 = 0$
$\therefore\ (y-14)(y-2) = 0$
$\therefore\ y-14 = 0$ or $y-2 = 0$
$\Rightarrow y = 14$ or $y = 2$
Hence the points are (0,14) and (0,2).

(iii) Let the abscissa of the point be x.
Then ordinate = 2x
So the coordinates of the point are (x,2x).
Since the point is at a distance of $\sqrt{10}$ from the point (4,3),
$\sqrt{[(4-x)^2+(3-2x)^2]} = \sqrt{10}$ [By distance formula]
Squaring both sides,
$(4-x)^2+(3-2x)^2 = 10$
$x^2+16-8x+4x^2 -12x+9-10 = 0$
$5x^2-20x+15 = 0$
Divide by 5
$x^2-4x+3 = 0$
$(x-3)(x-1) = 0$
$x-3 = 0$ or $x-1 = 0$
$x = 3$ or $x = 1$
So $2x = 2\times3 = 6$ or $2x = 2\times1 = 2$
Hence the points are (3,6) and (1,2).

5. Find the point on the x-axis which, is equidistant from the points (2, -5) and (-2, 9).

Solution:
Let the point on X axis be (x,0) which is equidistant from (2,-5) and (-2,9).
Distance between (x,0) and (2,-5) is equal to the distance between (x,0) and (-2,9).
$\therefore\ \sqrt{[(2-x)^2+(-5-0)^2]} = \sqrt{[(-2-x)^2+(9-0)^2]}$ [By distance formula]
$\therefore\ \sqrt{(4-4x+x^2+25)} = \sqrt{(4+4x+x^2+81)}$
$\therefore\ \sqrt{(x^2-4x+29)} = \sqrt{(x^2+4x+85)}$
Squaring both sides,
$x^2-4x+29 = x^2+4x+85$

$-4x-4x = 85-29$
$-8x = 56$
$\therefore x = 56/-8$
$\therefore x = -7$
Hence the point is (-7,0).

6. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, -1), (1, 3) and (x, 8) respectively.

Solution:
Coordinates of P are (6,-1).
Coordinates of Q are (1,3).
Coordinates of R are (x,8).
PQ = QR
$\therefore$ By distance formula, $\sqrt{[(1-6)^2+(3-(-1))^2]} = \sqrt{[(x-1)^2+(8-3)^2]}$
$\sqrt{[(-5)^2+(4^2]} = \sqrt{[(x-1)^2+(5)^2]}$
$\sqrt{[(25+16)]} = \sqrt{[x^2-2x+1+25]}$
$\sqrt{(41)} = \sqrt{[x^2-2x+26]}$
Squaring both sides,
$41= x^2-2x+26$
$\therefore\ x^2-2x+26-41 = 0$
$\therefore\ x^2-2x+15 = 0$
$\therefore (x+3)(x-5) = 0$
$(x+3)= 0$ or $(x-5) = 0$
$x = -3$ or $x = 5$
Hence the value of x is -3 or 5.

7. If Q(0, 1) is equidistant from P (5, -3) and R (x, 6) find the values of x.

Solution:
Q(0,1) is equidistant from P(5,-3) and R(x,6).
So PQ = QR
$\therefore$ By distance formula, $\sqrt{[(5-0)^2+(-3-1))^2]} = \sqrt{[(x-0)^2+(6-1)^2]}$
$\sqrt{[(5)^2+(-4^2]} = \sqrt{[(x^2+(5)^2]}$
$\sqrt{[(25+16)]} = \sqrt{[x^2+25]}$
$\sqrt{(41)} = \sqrt{[x^2+25]}$
Squaring both sides,
$41 = x^2+25$
$\therefore\ x^2+25-41 = 0$
$\therefore\ x^2-16= 0$
$\therefore (x-4)(x+4) = 0$
$\therefore (x-4) = 0$ or $(x+4) = 0$
$\Rightarrow x = 4$ or $x = -4$
Hence the value of x is 4 or -4.

8. Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Solution:
Let the point (7,1) be Q and the point (3,5) be R.

Let P(x,y) be the point equidistant from Q(7,1) and R(3,5).
So PQ = PR
∴By distance formula, $\sqrt{[(7-x)^2+(1-y))^2]} = \sqrt{[(3-x)^2+(5-y)^2]}$
∴$\sqrt{[x^2-14x+49+y^2-2y+1]} = \sqrt{[x^2-6x+9+y^2-10y+25]}$
$\sqrt{[x^2-14x+y^2-2y+50]} = \sqrt{[x^2-6x+y^2-10y+34]}$
Squaring both sides,
$x^2-14x+y^2-2y+50 = x^2-6x+y^2-10y+34$
$-14x+6x-2y+10y+50-34 = 0$
$-8x+8y+16 = 0$
Divide by 8
$-x+y+2 = 0$
$\Rightarrow y = x-2$
Hence the required relation is y = x-2.

9. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from the points Q (2, -5) and U (-3, 6), then find the coordinates of P.

Solution:
Let the y co-ordinate be x.
Then x coordinate is 2x.
So coordinates of P are (2x,x).
P is equidistant from the points Q (2, -5) and U (-3, 6).
∴PQ = PU
∴By distance formula, $\sqrt{[(2-2x)^2+(-5-x))^2]} = \sqrt{[(-3-2x)^2+(6-x)^2]}$
$\sqrt{[(4-8x+4x^2+25+10x+x^2]} = \sqrt{[9+12x+4x^2+36-12x+x^2]}$
$\sqrt{[29+2x+5x^2]} = \sqrt{[45+5x^2]}$
Squaring both sides,
$29+2x+5x^2 = 45+5x^2$
$2x+29-45 = 0$
$2x-16 = 0$
$2x = 16$
∴$x = 16/2$
∴$x = 8$
So $2x = 2\times8 = 16$
∴P(2x,x) = P(16,8)
Hence the coordinates of P are (16,8).

10. If the points A (4,3) and B (x, 5) are on a circle with centre C (2, 3), find the value of x.

Solution:
Given the points A(4,3) and B(x,5) are on the circle whose centre is C(2,3).
∴AC = BC [Radii of same circle]
∴By distance formula, $\sqrt{[(2-4)^2+(3-3)^2]} = \sqrt{[(2-x)^2+(3-5)^2]}$
$\sqrt{[(-2)^2+0]} = \sqrt{[4-4x+x^2+(-2)^2]}$
$\sqrt{4} = \sqrt{[4-4x+x^2+4]}$
$\sqrt{4} = \sqrt{[8-4x+x^2]}$
Squaring both sides,
$4 = 8-4x+x^2$
∴$x^2-4x+4 = 0$

$\Rightarrow (x-2)(x-2) = 0$
$\Rightarrow x = 2$
Hence the value of x is 2.

11. If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Solution:
Given A(0,2) is equidistant from B(3,p) and C(p,5)
$\therefore$ AB = AC
$\therefore$ By distance formula, $\sqrt{[(3-0)^2+(p-2)^2]} = \sqrt{[(p-0)^2+(5-2)^2]}$
$\sqrt{[(3)^2+(p-2)^2]} = \sqrt{[(p)^2+(3)^2]}$
$\sqrt{[9+p^2-4p+4]} = \sqrt{[p^2+9]}$
$\sqrt{[p^2-4p+13]} = \sqrt{[p^2+9]}$
Squaring both sides,
$p^2-4p+13 = p^2+9$
$-4p+13-9 = 0$
$-4p+4 = 0$
$-4p = -4$
$\therefore p = -4/-4 = 1$
Hence the value of p is 1.

12. Using distance formula, show that (3, 3) is the centre of the circle passing through the points (6, 2), (0, 4) and (4, 6).

Solution:
Let C(3, 3) is the centre of the circle passing through the points P(6, 2), Q(0, 4) and R(4, 6).
$\therefore$ CP = CQ = CR [radii of same circle]
By distance formula, CP = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore CP = \sqrt{[(6-3)^2+(2-3)^2]}$
$\therefore CP = \sqrt{[(3)^2+(-1)^2]}$
$\therefore CP = \sqrt{[9+1]}$
$\therefore CP = \sqrt{10}$
By distance formula, CQ = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore CQ = \sqrt{[(0-3)^2+(4-3)^2]}$
$\therefore CQ = \sqrt{[(3)^2+(1)^2]}$
$\therefore CQ = \sqrt{[9+1]}$
$\therefore CQ = \sqrt{10}$
By distance formula, CR = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore CR = \sqrt{[(4-3)^2+(6-3)^2]}$
$\therefore CR = \sqrt{[(1)^2+(3)^2]}$
$\therefore CR = \sqrt{[1+9]}$
$\therefore CR = \sqrt{10}$
Since CP = CQ = CR,
C(3,3) is the centre of the circle passing through the points P(6, 2), Q(0, 4) and R(4, 6).
Hence proved.

13. The centre of a circle is C(2α-1, 3α+1) and it passes through the point A (-3, -1). If a diameter of the circle is of length 20 units, find the value(s) of α.

Solution:

Centre of a circle is C(2α-1, 3α+1) and it passes through the point A (-3, -1).

Diameter of the circle = 20

$\therefore$ radius = 20/2= 10

$\therefore$ AC = 10 [radius]

By distance formula, AC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$\therefore 10 = \sqrt{[(2\alpha-1-(-3))^2+(3\alpha+1-(-1))^2]}$

$\therefore 10 = \sqrt{[(2\alpha-1+3)^2+(3\alpha+1+1)^2]}$

$\therefore 10 = \sqrt{[(2\alpha+2)^2+(3\alpha+2)^2]}$

Squaring both sides,

$\therefore 100 = [(2\alpha+2)^2+(3\alpha+2)^2]$

$\therefore 100 = 4\alpha^2+8\alpha+4+9\alpha^2+12\alpha+4$

$\therefore 100 = 13\alpha^2+20\alpha+8$

$\therefore 13\alpha^2+20\alpha+8-100 = 0$

$\therefore 13\alpha^2+20\alpha-92 = 0$

$\therefore 13\alpha^2-26\alpha+46\alpha-92 = 0$

$\therefore 13\alpha(\alpha-2)+46(\alpha-2) = 0$

$\therefore (\alpha-2)(13\alpha+46) = 0$

$\therefore \alpha-2 = 0$ or $13\alpha+46 = 0$

$\therefore \alpha = 2$ or $13\alpha = -46$

$\therefore \alpha = 2$ or $\alpha = -46/13$

Hence the value is $\alpha = 2$ or $\alpha = -46/13$.

14. Using distance formula, show that the points A (3, 1), B (6, 4) and C (8, 6) are collinear.

Solution:

Given points are A (3, 1), B (6, 4) and C (8, 6).

If AB+BC = AC, then the three points are collinear.

By distance formula, AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$\therefore AB = \sqrt{[(6-3)^2+(4-1)^2]}$

$\therefore AB = \sqrt{[(3)^2+(3)^2]}$

$\therefore AB = \sqrt{[9+9]}$

$\therefore AB = \sqrt{18}$

$\therefore AB = \sqrt{(9\times2)}$

$\therefore AB = 3\sqrt{2}$

By distance formula, BC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$\therefore BC = \sqrt{[(8-6)^2+(6-4)^2]}$

$\therefore BC = \sqrt{[(2)^2+(2)^2]}$

$\therefore BC = \sqrt{[4+4]}$

$\therefore BC = \sqrt{8}$

$\therefore BC = \sqrt{(4\times2)}$

$\therefore BC = 2\sqrt{2}$

By distance formula, AC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$\therefore AC = \sqrt{[(8-3)^2+(6-1)^2]}$

$\therefore AC = \sqrt{[(5)^2+(5)^2]}$

$\therefore AC = \sqrt{[25+25]}$

$\therefore AC = \sqrt{50}$

$\therefore AC = \sqrt{(25\times2)}$

$\therefore AC = 5\sqrt{2}$
$AB+BC = 3\sqrt{2}+ 2\sqrt{2} = 5\sqrt{2} = AC$
Hence proved.
So A, B, C are collinear.

15. Check whether the points (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Solution:
Let A(5, -2), B(6, 4) and C(7, -2) are the vertices of an isosceles triangle.
By distance formula, $AB = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AB = \sqrt{[(6-5)^2+(4-(-2))^2]}$
$\therefore AB = \sqrt{[(1)^2+(4+2)^2]}$
$\therefore AB = \sqrt{[1+6^2]}$
$\therefore AB = \sqrt{(1+36)}$
$\therefore AB = \sqrt{37}$
By distance formula, $AC = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AC = \sqrt{[(7-5)^2+(-2-(-2))^2]}$
$\therefore AC = \sqrt{[(2)^2+(-2+2)^2]}$
$\therefore AC= \sqrt{[4+0]}$
$\therefore AC = \sqrt{4}$
$\therefore AC = 2$
By distance formula, $BC = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore BC = \sqrt{[(7-6)^2+(-2-4)^2]}$
$\therefore BC = \sqrt{[(1)^2+(-6)^2]}$
$\therefore BC= \sqrt{[1+36]}$
$\therefore BC = \sqrt{37}$
$\therefore BC = \sqrt{37}$
Here AB = BC.
Hence $\triangle$ ABC is an isosceles triangle.

16. Name the type of triangle formed by the points A (-5, 6), B (-4, -2) and (7, 5).

Solution:
The three vertices of the triangle are A (-5, 6), B (-4, -2) and (7, 5).
By distance formula, $AB = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AB = \sqrt{[(-4-(-5))^2+(4-(-2-6))^2]}$
$\therefore AB = \sqrt{[(1)^2+(-8)^2]}$
$\therefore AB = \sqrt{[1+64]}$
$\therefore AB = \sqrt{65}$
By distance formula, $AC = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AC = \sqrt{[(7-(-5))^2+(5-6)^2]}$
$\therefore AC = \sqrt{[(12)^2+(-1)^2]}$
$\therefore AC= \sqrt{[144+1]}$
$\therefore AC = \sqrt{145}$
By distance formula, $BC = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore BC = \sqrt{[(7-(-4))^2+(5-(-2))^2]}$
$\therefore BC = \sqrt{[(11)^2+(7)^2]}$
$\therefore BC= \sqrt{[121+49]}$

$\therefore$ BC $= \sqrt{170}$
Length of all sides of the triangle are different.
So $\triangle$ABC is a scalene triangle.

17. Show that the points (1, 1), (- 1, – 1) and ($-\sqrt{3}, \sqrt{3}$) form an equilateral triangle.

Solution:
Let A(1,1), B(-1,-1) and C($-\sqrt{3}$, $\sqrt{3}$) be the vertices of $\triangle$ABC.
By distance formula, AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ AB $= \sqrt{[(-1-1)^2+(-1-1)^2]}$
$\therefore$ AB $= \sqrt{[(-2)^2+(-2)^2]}$
$\therefore$ AB $= \sqrt{[4+4]}$
$\therefore$ AB $= \sqrt{8}$
By distance formula, BC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ BC $= \sqrt{[(-\sqrt{3}-(-1))^2+(\sqrt{3}-(-1))^2]}$
$\therefore$ BC $= \sqrt{[(-\sqrt{3}+1)^2+(\sqrt{3}+1)^2]}$
$\therefore$ BC $= \sqrt{[3-2\sqrt{3}+1+3+2\sqrt{3}+1]}$
$\therefore$ BC $= \sqrt{8}$
By distance formula, AC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ AC $= \sqrt{[(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2]}$
$\therefore$ AC $= \sqrt{[3+2\sqrt{3}+1+3-2\sqrt{3}+1]}$
$\therefore$ AC$= \sqrt{8}$
Here AB = BC = AC.
So the points form an equilateral triangle.

18. Show that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Solution:
Let A(7,10), B(-2,5) and C(3,-4) be the vertices of $\triangle$ABC.
By distance formula, AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ AB $= \sqrt{[(-2-7)^2+(5-10)^2]}$
$\therefore$ AB $= \sqrt{[(-9)^2+(-5)^2]}$
$\therefore$ AB $= \sqrt{[81+25]}$
$\therefore$ AB $= \sqrt{106}$
By distance formula, BC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ BC $= \sqrt{[(3-(-2))^2+(-4-5)^2]}$
$\therefore$ BC $= \sqrt{[(5)^2+(-9)^2]}$
$\therefore$ BC $= \sqrt{(25+81)}$
$\therefore$ BC $= \sqrt{106}$
By distance formula, AC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ AC $= \sqrt{[(3-7)^2+(-4-10)^2]}$
$\therefore$ AC $= \sqrt{[(-4)^2+(-14)^2]}$
$\therefore$ AC$= \sqrt{(16+196)}$
$\therefore$ AC$= \sqrt{212}$
Here AB = BC.
So $\triangle$ABC is an isosceles triangle.
$AB^2+BC^2 = 106+106$
$AB^2+BC^2 = 212 = AC^2$ [Pythagoras theorem]
So $\triangle$ABC is a right triangle.

Hence △ABC is an isosceles right triangle.

19. The points A (0, 3), B (- 2, a) and C (- 1, 4) are the vertices of a right angled triangle at A, find the value of a.

Solution:
Given the points A (0, 3), B (- 2, a) and C (- 1, 4) are the vertices of a right angled triangle at A.
By distance formula, AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AB = \sqrt{[(-2-0)^2+(a-3)^2]}$
$\therefore AB = \sqrt{(4+a^2-6a+9)}$
$\therefore AB = \sqrt{(a^2-6a+13)}$
By distance formula, BC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore BC = \sqrt{[(-1-(-2))^2+(4-a)^2]}$
$\therefore BC = \sqrt{[(1)^2+16-8a+a^2]}$
$\therefore BC = \sqrt{(17-8a+a^2)}$
$\therefore BC = \sqrt{(a^2-8a+17)}$
By distance formula, AC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AC = \sqrt{[(-1-0)^2+(4-3)^2]}$
$\therefore AC = \sqrt{[(-1)^2+(1)^2]}$
$\therefore AC= \sqrt{(1+1)}$
$\therefore AC= \sqrt{2}$
$BC^2 = AC^2 + AB^2$ [Pythagoras theorem]
$\therefore a^2-8a+17 = 2+a^2-6a+13$
$-8a+6a+17-2-13 = 0$
$-2a+2 = 0$
$\Rightarrow 2a = 2$
$\Rightarrow a = 2/2 = 1$
Hence the value of a is 1.

20. Show that the points (0, – 1), (- 2, 3), (6, 7) and (8, 3), taken in order, are the vertices of a rectangle. Also find its area.

Solution:
Let the points A(0, – 1), B(- 2, 3), C(6, 7) and D(8, 3) be the vertices of a rectangle.
By distance formula, AB = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore AB = \sqrt{[(-2-0)^2+(3-(-1))^2]}$
$\therefore AB = \sqrt{(-2)^2+(4)^2}$
$\therefore AB = \sqrt{(4+16)}$
$\therefore AB = \sqrt{20}$
$\therefore AB = \sqrt{(4\times5)}$
$\therefore AB = 2\sqrt{5}$
By distance formula, BC = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore BC = \sqrt{[(6-(-2))^2+(7-3)^2]}$
$\therefore BC = \sqrt{[(8)^2+4^2]}$
$\therefore BC = \sqrt{(64+16)}$
$\therefore BC = \sqrt{(80)}$
$\therefore BC = \sqrt{(5\times16)}$
$\therefore BC = 4\sqrt{5}$

By distance formula, CD = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ CD = $\sqrt{[(8-6)^2+(3-7)^2]}$
$\therefore$ CD = $\sqrt{[(2)^2+(-4)^2]}$
$\therefore$ CD = $\sqrt{(4+16)}$
$\therefore$ CD = $\sqrt{(20)}$
$\therefore$ CD = $\sqrt{(4\times5)}$
$\therefore$ CD = $2\sqrt{5}$
By distance formula, AD = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ AD = $\sqrt{[(8-0)^2+(3-(-1))^2]}$
$\therefore$ AD = $\sqrt{[(8)^2+(4)^2]}$
$\therefore$ AD = $\sqrt{(64+16)}$
$\therefore$ AD = $\sqrt{(80)}$
$\therefore$ AD = $\sqrt{(5\times16)}$
$\therefore$ AD = $4\sqrt{5}$
Here AB = CD and BC = AD.
Hence these are the vertices of a rectangle.
Area of □ABCD = AB×BC
= $2\sqrt{5}\times4\sqrt{5}$
= 40 sq. units.
Hence the area of □ABCD is 40 sq. units.

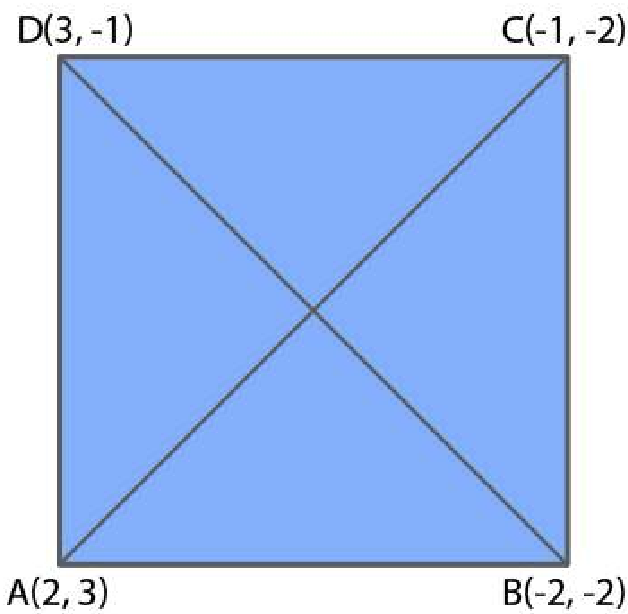
21. If P (2, -1), Q (3, 4), R (-2, 3) and S (-3, -2) be four points in a plane, show that PQRS is a rhombus but not a square. Find the area of the rhombus.

Solution:
Given P (2, -1), Q (3, 4), R (-2, 3) and S (-3, -2) be four points in a plane.
By distance formula, PQ = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ PQ = $\sqrt{[(3-2)^2+(4-(-1))^2]}$
$\therefore$ PQ = $\sqrt{(1)^2+(5)^2}$
$\therefore$ PQ = $\sqrt{(1+25)}$
$\therefore$ PQ = $\sqrt{26}$
By distance formula, QR = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ QR = $\sqrt{[(-2-3)^2+(3-4)^2]}$
$\therefore$ QR = $\sqrt{[(-5)^2+(-1)^2]}$
$\therefore$ QR = $\sqrt{[25+1]}$
$\therefore$ QR = $\sqrt{26}$
By distance formula, RS = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ RS = $\sqrt{[(-3-(-2))^2+(-2-3)^2]}$
$\therefore$ RS = $\sqrt{[(-1)^2+(-5)^2]}$
$\therefore$ RS = $\sqrt{[1+25]}$
$\therefore$ RS = $\sqrt{26}$
By distance formula, PS = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ PS = $\sqrt{[(-3-2)^2+(-2-(-1))^2]}$
$\therefore$ PS = $\sqrt{[(-5)^2+(-1)^2]}$
$\therefore$ PS = $\sqrt{[25+1]}$
$\therefore$ PS = $\sqrt{26}$
Here PQ = QR = RS = PS.
So it can be a rhombus or a square.

Diagonal, PR = $\sqrt{[(-2-2)^2+(3-(-1))^2]}$ [Distance formula]
PR = $\sqrt{[(-4))^2+(4)^2]}$
PR = $\sqrt{(16+16)} = \sqrt{32} = \sqrt{(16\times2)} = 4\sqrt{2}$
Diagonal, QS = $\sqrt{[(-3-3)^2+(-2-4)^2]}$ [Distance formula]
QS = $\sqrt{[(-6))^2+(-6)^2]}$
QS = $\sqrt{[36+36]} = \sqrt{(2\times36)} = 6\sqrt{2}$
Here diagonals are not equal. So PQRS is not a square. It is a rhombus.
Area of rhombus PQRS = ½ ×PR×QS
= ½ × $4\sqrt{2}\times6\sqrt{2}$
= 24 sq units.
Hence the area of the rhombus PQRS is 24 sq. units.

22. Prove that the points A (2, 3), B (-2, 2), C (-1, -2) and D (3, -1) are the vertices of a square ABCD.

Solution:
Let A (2, 3), B (-2, 2), C (-1, -2) and D (3, -1) are the vertices of a square ABCD.

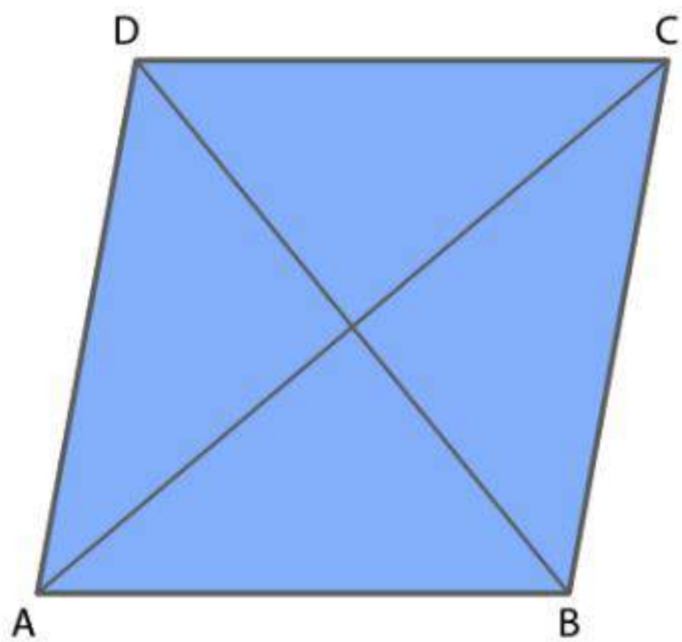


Using distance formula, we find the length of the sides and length of the diagonals.
AB = $\sqrt{[(-2-2)^2+(2-3)^2]}$
= $\sqrt{[(-4)^2+(-1)^2]}$
= $\sqrt{[(16+1)]}$
= $\sqrt{17}$
BC = $\sqrt{[(-2-(-1))^2+(2-(-2))^2]}$
= $\sqrt{[(-1)^2+(4)^2]}$
= $\sqrt{[(1+16)]}$
= $\sqrt{17}$
CD = $\sqrt{[(3-(-1))^2+(-1-(-2))^2]}$
= $\sqrt{[(4)^2+(1)^2]}$
= $\sqrt{[(16+1)]}$
= $\sqrt{17}$

AD = $\sqrt{[(3-2)^2+(-1-3)^2]}$
= $\sqrt{[(1)^2+(-4)^2]}$
= $\sqrt{[(1+16)]}$
= $\sqrt{17}$
Here AB = BC = CD = AD.
All the sides are equal .
Diagonal AC = $\sqrt{[(-1-2)^2+(-2-3)^2]}$
= $\sqrt{[(-3)^2+(-5)^2]}$
= $\sqrt{[9+25]}$
= $\sqrt{34}$
Diagonal BD = $\sqrt{[(3-(-2))^2+(-1-2)^2]}$
$\sqrt{[(5)^2+(-3)^2]}$
= $\sqrt{[(25+9)]}$
= $\sqrt{34}$
AC = BD
So diagonals are also equal.
Hence the points are the vertices of a square.

23. Name the type of quadrilateral formed by the following points and give reasons for your answer :
(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)
(ii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:



(i)Let A(-1, -2), B(1, 0), C(-1, 2), D(-3, 0) are the given points.
Using distance formula, we find the length of the sides and length of the diagonal.
AB = $\sqrt{[(1-(-1))^2+(0-(-2))^2]}$
= $\sqrt{[(2)^2+(2)^2]}$
= $\sqrt{(4+4)}$
= $\sqrt{8}$
BC = $\sqrt{[(-1-1))^2+(2-0)^2]}$

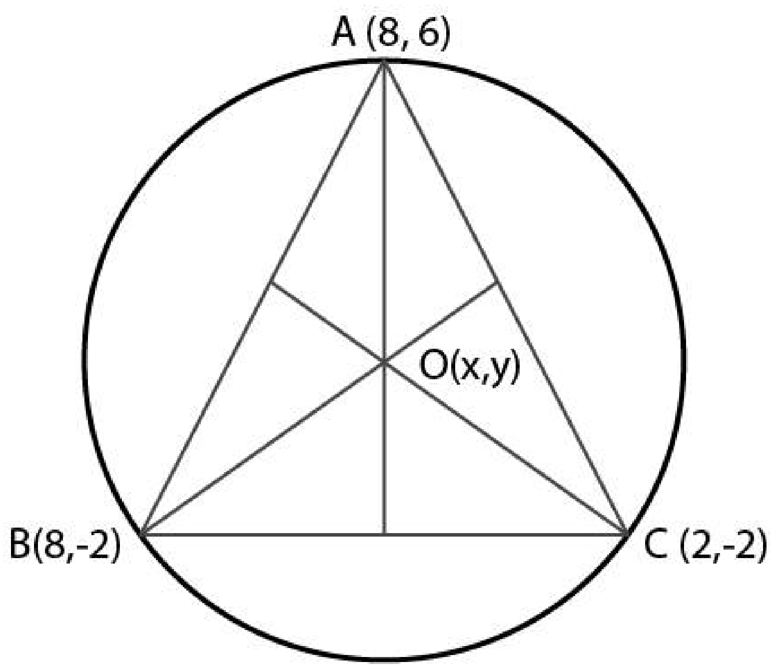
$= \sqrt{[(-2)^2+(2)^2]}$
$= \sqrt{(4+4)}$
$= \sqrt{8}$
$CD = \sqrt{[(-3-(-1))^2+(0-2)^2]}$
$= \sqrt{[(-2)^2+(-2)^2]}$
$= \sqrt{(4+4)}$
$= \sqrt{8}$
$AD = \sqrt{[(-3-(-1))^2+(0-(-2))^2]}$
$= \sqrt{[(-2)^2+(2)^2]}$
$= \sqrt{(4+4)}$
$= \sqrt{8}$
Diagonal $AC = \sqrt{[(-1-(-1))^2+(2-(-2))^2]}$
$= \sqrt{[(0)^2+(4)^2]}$
$= \sqrt{[16]}$
$= 4$
Diagonal $BD = \sqrt{[(-3-1)^2+(0-0)^2]}$
$\sqrt{(-4)^2+0}$
$= \sqrt{16}$
$= 4$
AC = BD
So diagonals are also equal.
Also AB = BC = CD = AD.
∴ All the sides are equal .
Hence quadrilateral ABCD is a square.

(ii) Let A(4, 5), B(7, 6), C(4, 3), D(1, 2) are the given points.
Using distance formula, we find the length of the sides and length of the diagonal.
$AB = \sqrt{[(7-4)^2+(6-5)^2]}$
$= \sqrt{[(3)^2+(1)^2]}$
$= \sqrt{(9+1)}$
$= \sqrt{10}$
$BC = \sqrt{[(4-7)^2+(3-6)^2]}$
$= \sqrt{[(-3)^2+(-3)^2]}$
$= \sqrt{(9+9)}$
$= \sqrt{18}$
$CD = \sqrt{[(1-4)^2+(2-3)^2]}$
$= \sqrt{[(-3)^2+(-1)^2]}$
$= \sqrt{(9+1)}$
$= \sqrt{10}$
$AD = \sqrt{[(1-4)^2+(2-5)^2]}$
$= \sqrt{[(-3)^2+(-3)^2]}$
$= \sqrt{(9+9)}$
$= \sqrt{18}$
Diagonal $AC = \sqrt{[(4-4)^2+(3-5)^2]}$
$= \sqrt{[(0)^2+(-2)^2]}$
$= \sqrt{4}$
$= 2$
Diagonal $BD = \sqrt{[(1-7)^2+(2-6)^2]}$
$\sqrt{(-6)^2+-4^2}$
$= \sqrt{(36+16)}$

$= \sqrt{52}$
$AC \neq BD$
So diagonals are not equal.
AB = CD
BC = AD.
$\therefore$ Opposite sides are equal .
Since opposite sides are equal and diagonals are not equal, ABCD is a parallelogram.

24. Find the coordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, -2) and (2, -2). Also, find its circumradius.

Solution:



Let O(x,y) be the circum centre of the circle.
Let A(8, 6), B(8, -2) and C(2, -2) be the vertices of the triangle.
OB = OC [Radii of same circle]
By distance formula,
$\therefore \sqrt{[(8-x)^2+(-2-y)^2]} = \sqrt{[(2-x)^2+(-2-y)^2]}$
Squaring both sides,
$(8-x)^2+(-2-y)^2 = (2-x)^2+(-2-y)^2$
$64+x^2-16x+4+4y+y^2 = 4-4x+x^2+4+4y+y^2$
$64-16x = 4-4x$
$\Rightarrow 12x = 60$
$\Rightarrow x = 60/12 = 5$

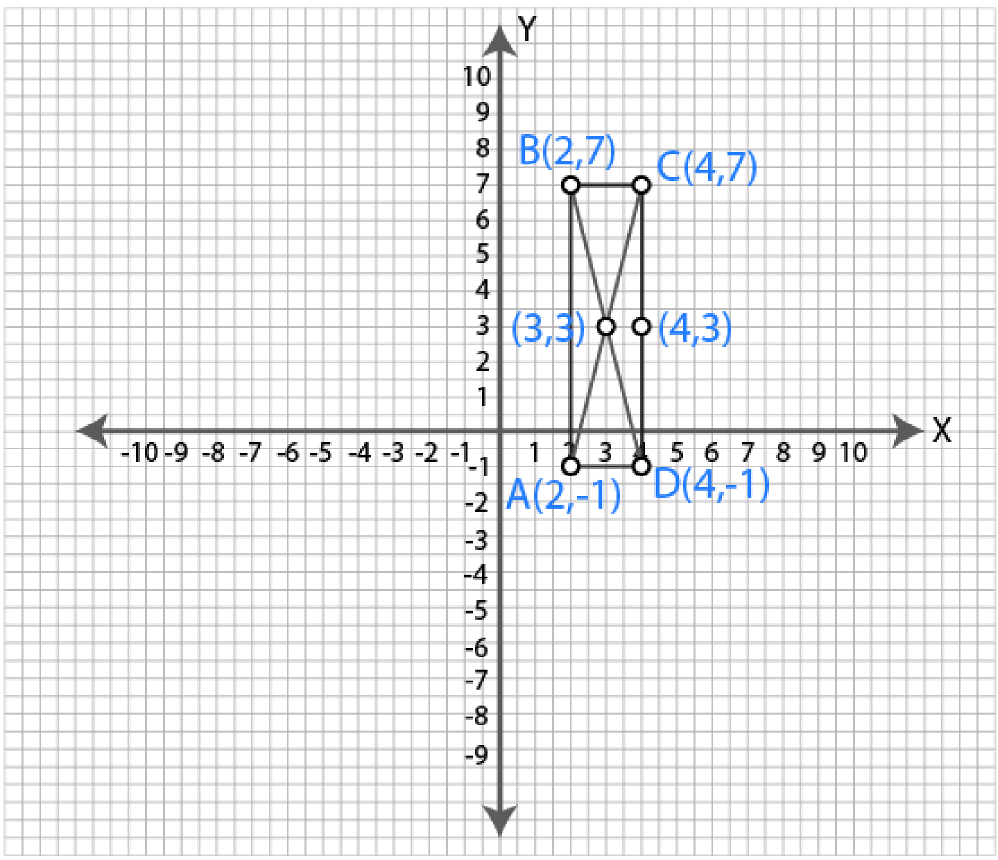
OA = OB [Radii of same circle]
By distance formula,
$\therefore \sqrt{[(8-x)^2+(6-y)^2]} = \sqrt{[(8-x)^2+(-2-y)^2]}$

Squaring both sides,
$(8-x)^2+(6-y)^2= (8-x)^2+(-2-y)^2$
$36-12y+y^2 = 4+4y+y^2$
$-12y-4y = 4-36$
$\Rightarrow -16y = -32$
$\Rightarrow y = 32/16 = 2$
Hence the coordinates of O are (5,2).
$OA = \sqrt{[(8-5)^2+(6-2)^2]}$
$= \sqrt{[(3)^2+(4)^2]}$
$= \sqrt{[9+16]}$
$= \sqrt{25} = 5$
Hence the circumradius is 5 units.

CHAPTER TEST

1. Three vertices of a rectangle are A (2, -1), B (2, 7) and C(4, 7). Plot these points on a graph and hence use it to find the co-ordinates of the fourth vertex D
Also find the co-ordinates of
(i) the mid-point of BC
(ii) the mid-point of CD
(iii) the point of intersection of the diagonals.
What is the area of the rectangle ?

Solution:
Given three vertices of a rectangle are A (2, -1), B (2, 7) and C(4, 7).
These points are marked on the graph shown below.
Join the points to form rectangle ABCD.
Also join the diagonals AC and BD.

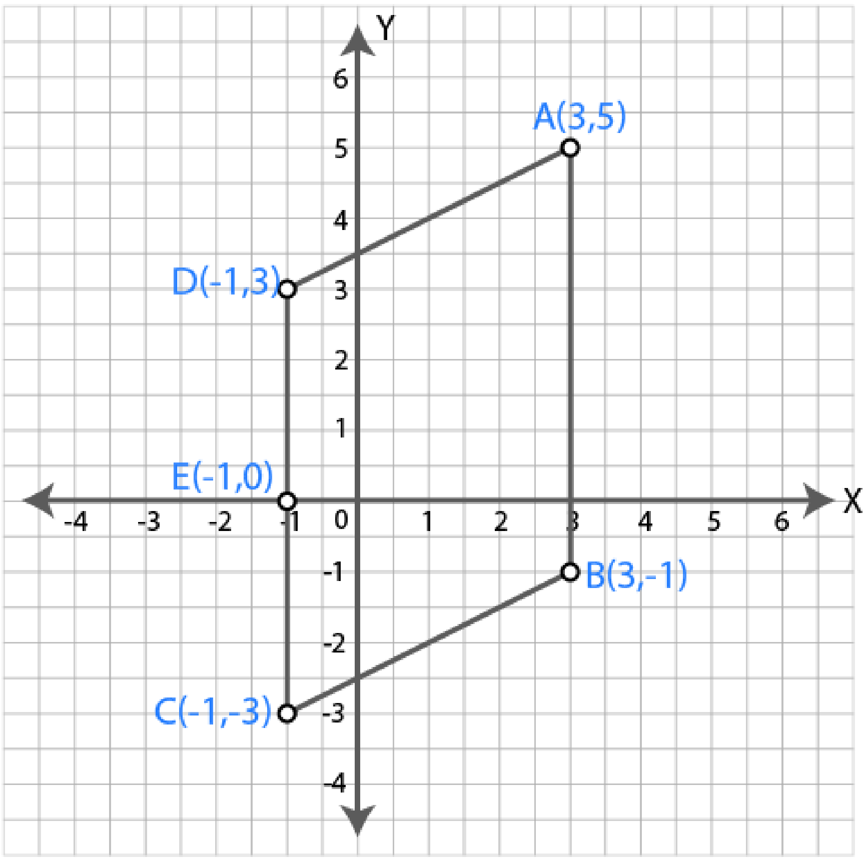


The coordinates of fourth vertex D is (4,-1).
(i) The midpoint of BC is (3,7).
(ii) The midpoint of CD is (4,3).
(iii)The point of intersection of diagonals is (3,3).
Area of the rectangle ABCD = AB×BC

= 8×2
= 16 sq. units.
Hence the area of the rectangle is 16 sq. units.

2. Three vertices of a parallelogram are A (3, 5), B (3, -1) and C (-1, -3). Plot these points on a graph paper and hence use it to find the coordinates of the fourth vertex D. Also find the coordinates of the mid-point of the side CD. What is the area of the parallelogram?

Solution:
Given A (3, 5), B (3, -1) and C (-1, -3) are the three vertices of a parallelogram.
These points are marked on the graph shown below.
Join the points to form parallelogram ABCD.



The coordinates of fourth vertex D is (-1,3).
The coordinates of midpoint of CD is (-1,0).
Area of parallelogram ABCD = Base ×height
= AB×EF
= 6×4
= 24 sq. units.
Hence the area of the parallelogram is 24 sq. units.

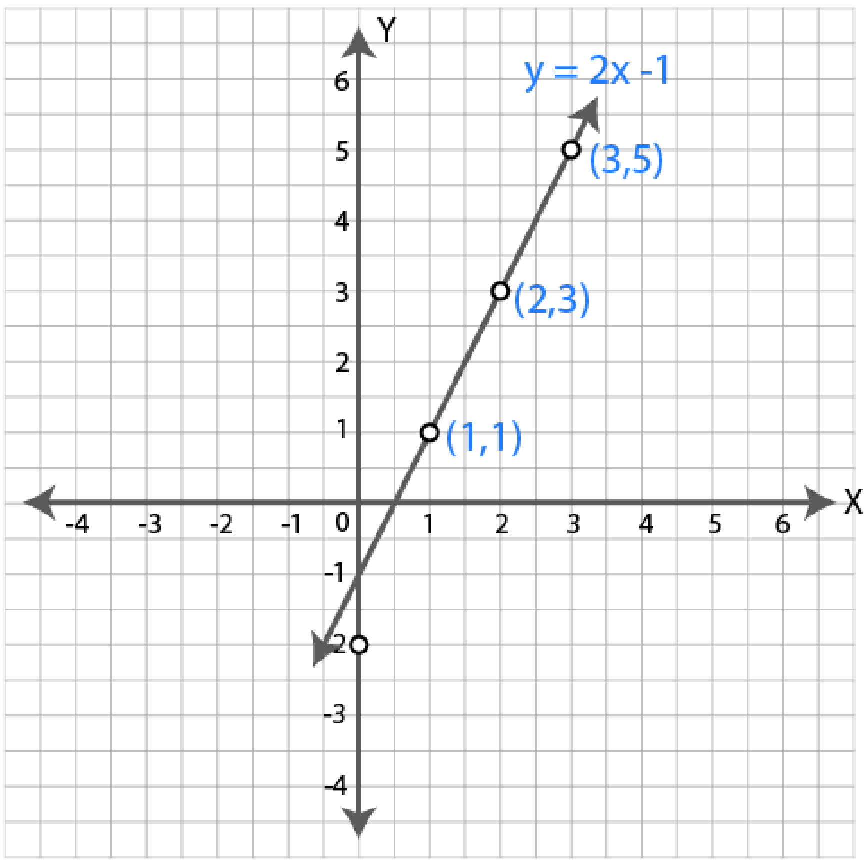
3. Draw the graphs of the following linear equations.

(i) y = 2x – 1
(ii) 2x + 3y = 6
(iii) 2x – 3y = 4.
Also find slope and y-intercept of these lines.

Solution:
(i)y = 2x-1
when x = 1, y = 2×1-1 = 1
when x = 2, y = 2×2-1 = 4-1 = 3
when x = 3, y = 2×3-1 = 6-1 = 5

x	1	2	3
y	1	3	5

Mark the above points on graph. Join them.



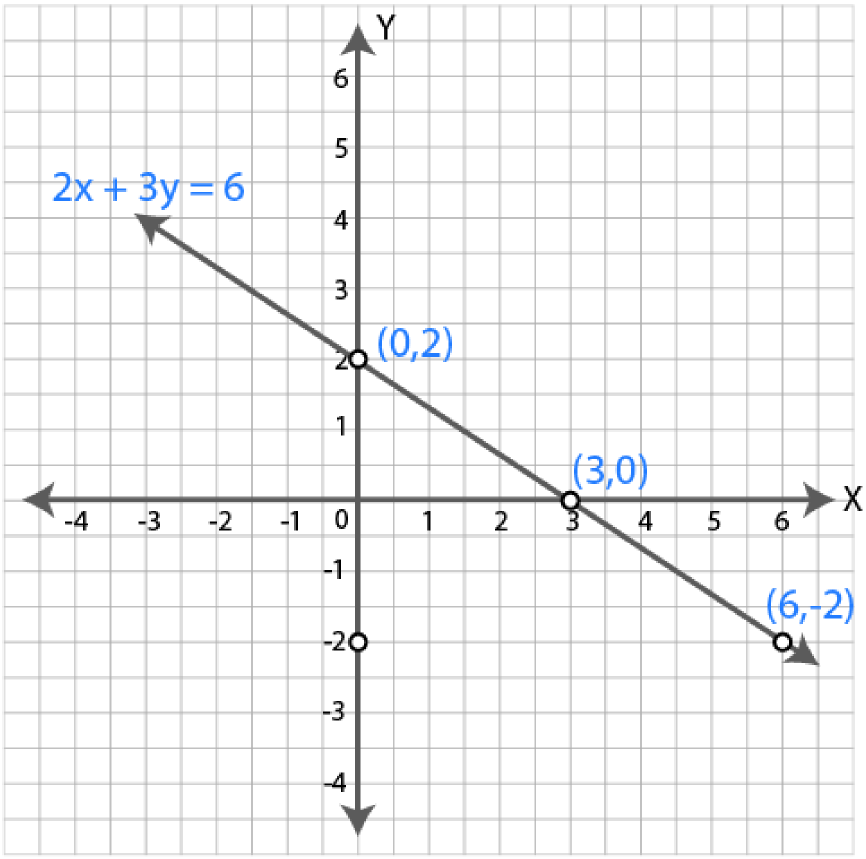
Slope of the line y = mx+c is m.
y intercept is c.
∴Slope of the line y = 2x-1 is m = 2.
Y intercept c = -1
Hence the slope is 2 and y intercept is -1.

(ii)2x+3y = 6

$\Rightarrow 3y = 6-2x$
$\Rightarrow y = (6-2x)/3$
when $x = 0$, $y = (6-2\times0)/3 = 6/3 = 2$
when $x = 3$, $y = (6-2\times3)/3 = 0$
when $x = 6$, $y = (6-2\times6)/3 = -6/3 = -2$

x	1	2	3
y	1	3	5

Mark the above points on graph. Join them.

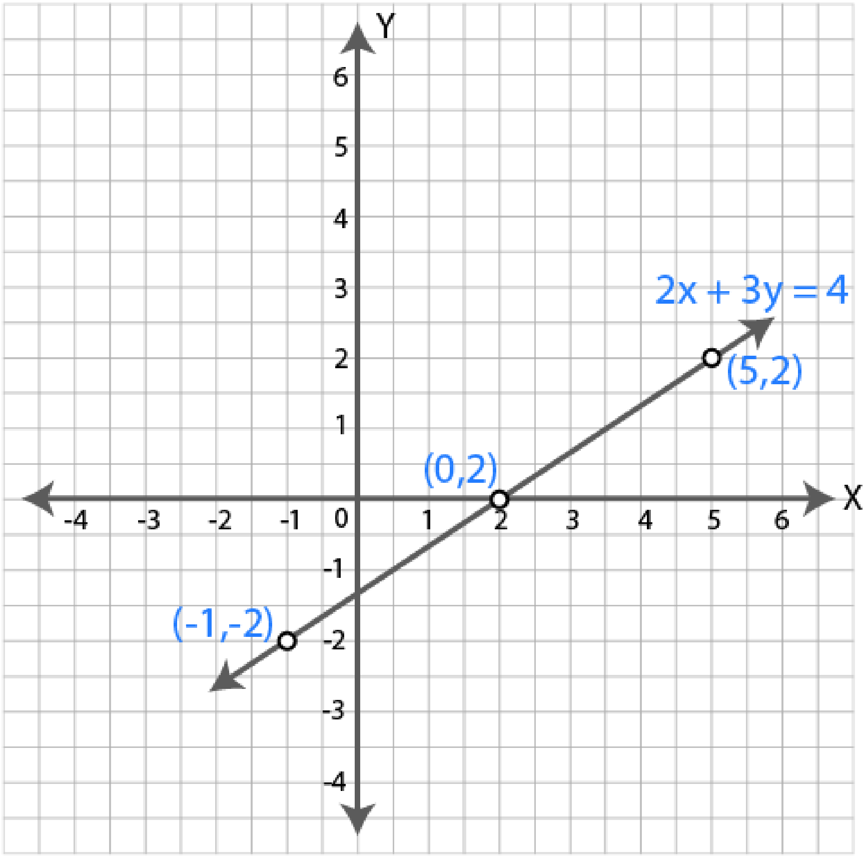


Slope of the line $y = mx+c$ is m.
y intercept is c.
$\therefore$ Slope of the line $y = (6-2x)/3$ is $m = -2/3$.
Y intercept $c = 6/3 = 2$
Hence the slope is $-2/3$ and y intercept is 2.

(iii) $2x-3y = 4$
$\Rightarrow 3y = 2x-4$
$\Rightarrow y = (2x-4)/3$
$\Rightarrow y = (2/3)x-4/3$
When $x = 2$, $y = (2\times2-4)/3 = 0$
When $x = 5$, $y = (2\times5-4)/3 = (10-4)/3 = 6/3 = 2$
When $x = -1$, $y = (2\times-1-4)/3 = -6/3 = -2$

x	2	5	-1
y	0	2	-2

Mark the above points on graph. Join them.



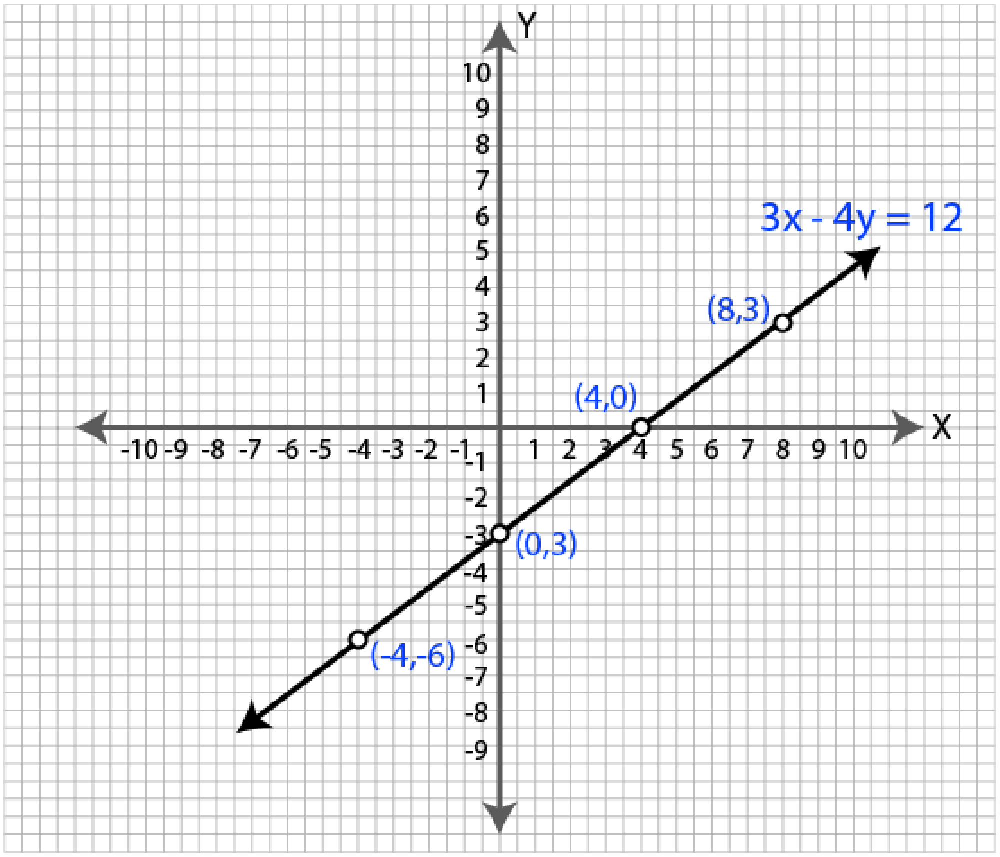
Slope of the line y = mx+c is m.
y intercept is c.
∴Slope of the line y = (2/3)x-4/3 is m = 2/3.
Y intercept c = -4/3
Hence the slope is 2/3 and y intercept is -4/3.

4. Draw the graph of the equation 3x – 4y = 12. From the graph, find :
(i) the value of y when x = -4
(ii) the value of x when y = 3.

Solution:
3x-4y = 12 …(i)
⇒4y = 3x-12
⇒y = (3x-12)/4
When x = 0, y = (3×0-12)/4 = -12/4 = -3
When x = 4, y = (3×4-12)/4 = (12-12)/4 = 0
When x = 8, y = (3×8-12)/4 = (24-12)/4 = 12/4 = 3

x	0	4	8
y	-3	0	3

Mark the above points on graph. Join them.



(i) When x = -4, the value of y is -6.
(ii) When y = 3, the value of x is 8.

5. Solve graphically, the simultaneous equations: 2x – 3y = 7; x + 6y = 11.

Solution:
2x-3y = 7 ..(i)
⇒3y = 2x-7
⇒y = (2x-7)/3
When x = -1, y = (2×-1-7)/3 = -9/3 = -3
When x = 2, y = (2×2-7)/3 = -3/3 = -1
When x = 5, y = (2×5-7)/3 = 3/3 = 1

x	0	4	8
y	-3	0	3

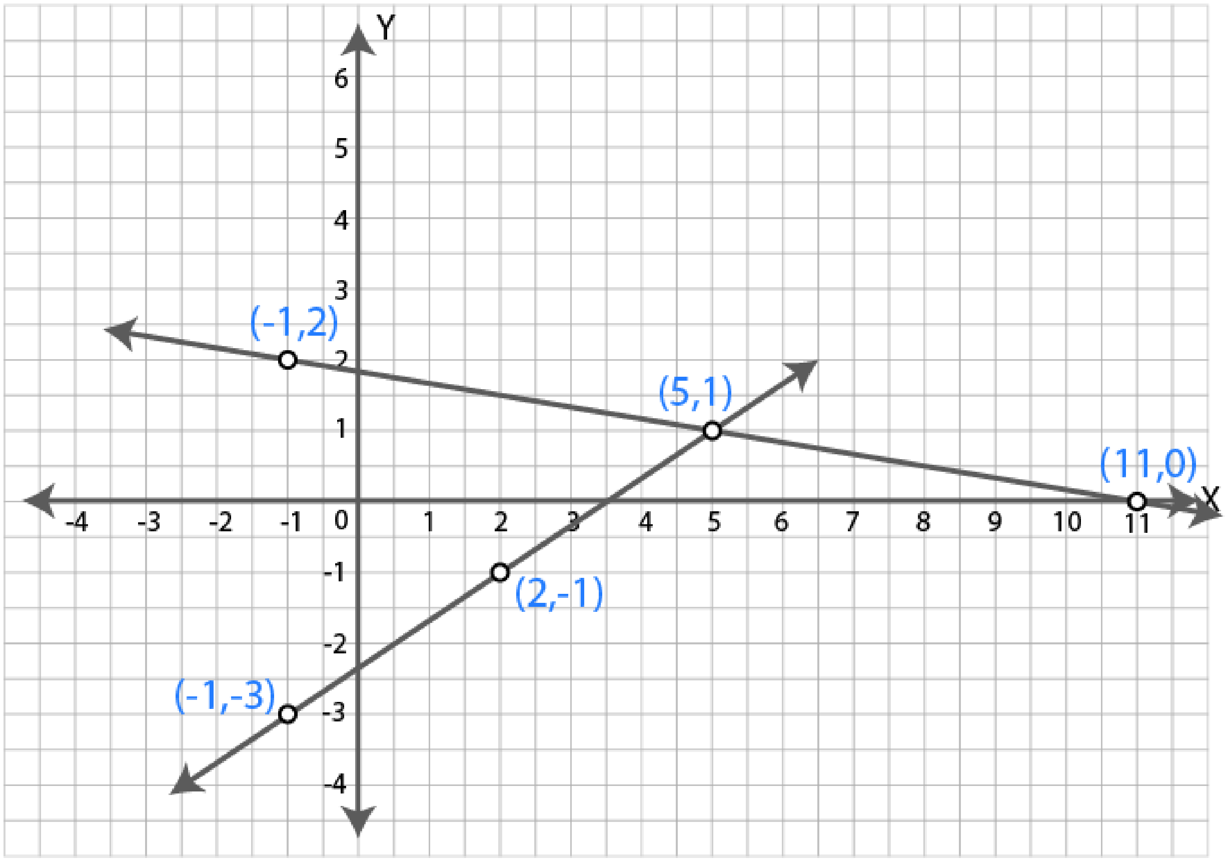
Mark the above points on graph. Join them.

x+6y = 11 …(ii)
⇒6y = 11-x

⇒y = (11-x)/6
When x = -1, y = (11-(-1))/6 = 12/6 = 2
When x = 5, y = (11-5)/6 = 6/6 = 1
When x = 11, y = (11-11)/6 = 0

x	-1	5	11
y	2	1	0

Mark the above points on graph. Join them.



From the graph , it is clear that the two lines intersect at (5,1).
Hence x= 5 and y = 1.

6. Solve the following system of equations graphically: x – 2y – 4 = 0, 2x + y – 3 =0.
Solution:

It is given that
x – 2y – 4 = 0, 2x + y – 3 =0

x – 2y – 4 = 0
It can be written as
x = 2y + 4

By giving different values to y, we obtain the corresponding values of x

x	4	2	0
y	0	-1	-2

Now plot the points (4, 0), (2, -1) and (0, -2) on the graph and join them to obtain a line.

$2x + y - 3 = 0$

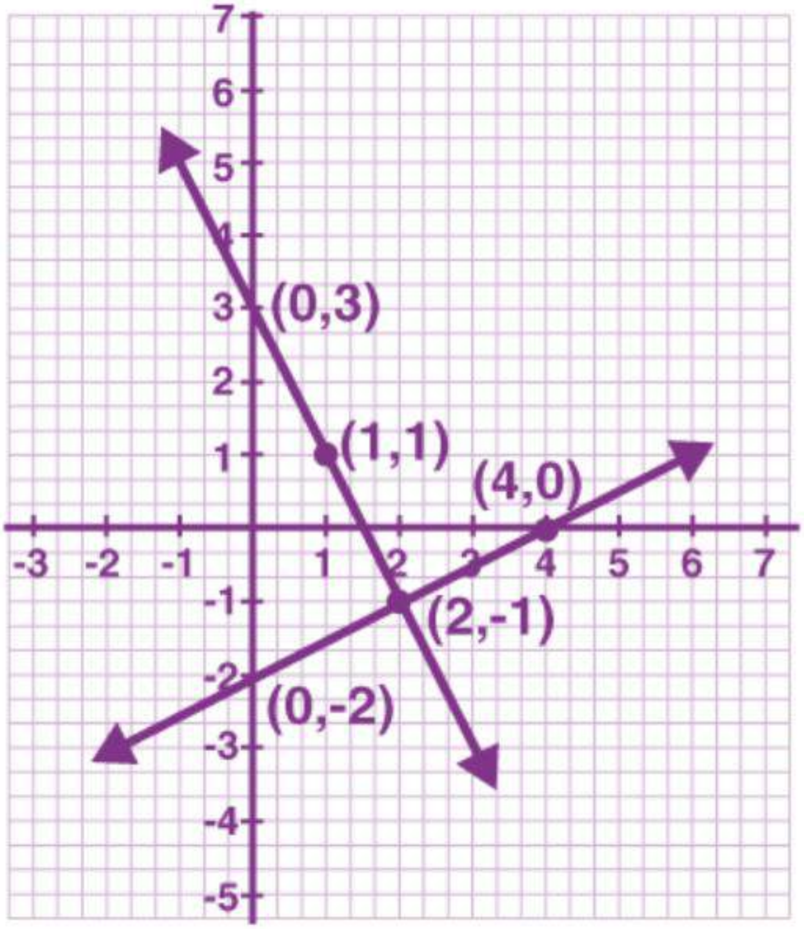
It can be written as

$y = 3 - 2x$

x	0	1	2
y	3	1	-1

Now plot the points (0, 3), (1, 1) and (2, -1) on the graph and join them to obtain another line.

Both the lines intersect each other at x = 2 and y = - 1.



7. Using a scale of 1 cm to 1 unit for both the axes, draw the graphs of the following equations: 6y = 5x + 10, y = 5x – 15. From the graph, find

(i) the coordinates of the point where the two lines intersect.

(ii) the area of the triangle between the lines and the x-axis.

Solution:

It is given that

6y = 5x + 10 and y = 5x – 15

6y = 5x + 10
It can be written as
y = (5x + 10)/ 6
By giving different values to x, we obtain the corresponding values of y

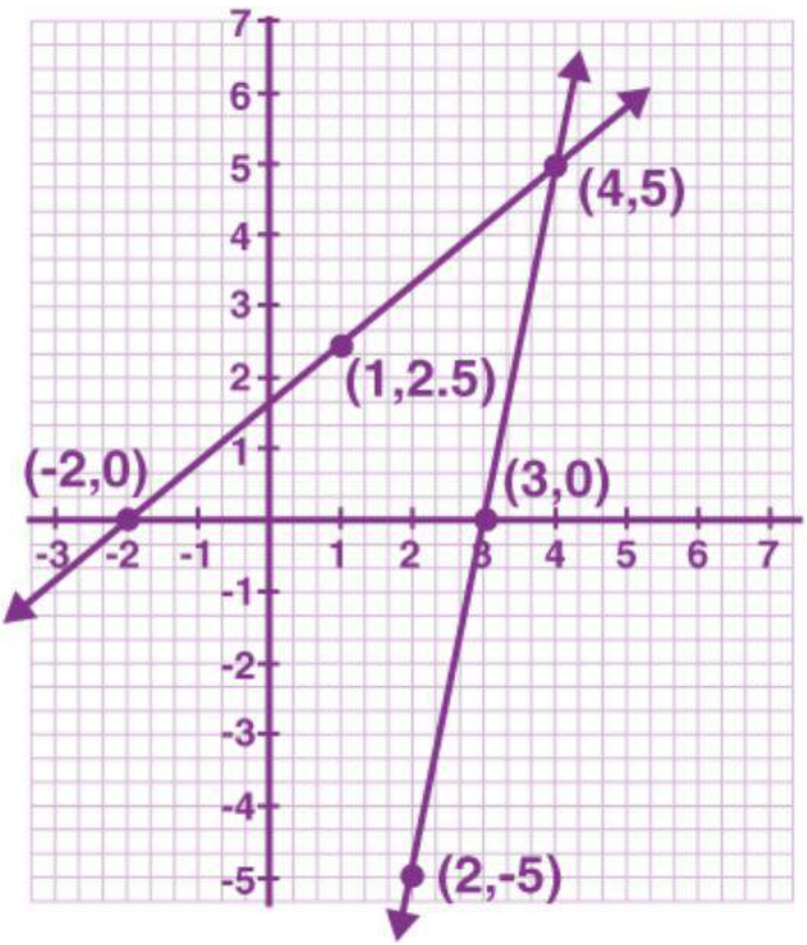
x	1	-2	4
y	2.5	0	5

Now plot the points (1, 2.5), (-2, 0) and (4, 5) on the graph and join them to obtain a line.

y = 5x – 15

x	2	3	4
y	-5	0	5

Now plot the points (2, -5), (3, 0) and (4, 5) on the graph and join them to obtain another line.
Both the lines intersect each other at x = 4 and y = 5.



8. Find, graphically, the coordinates of the vertices of the triangle formed by the lines:
8y – 3x + 7 = 0, 2x – y + 4 = 0 and 5x + 4y = 29.
Solution:

It is given that
8y – 3x + 7 = 0, 2x – y + 4 = 0 and 5x + 4y = 29

8y – 3x + 7 = 0
It can be written as
8y = 3x – 7
y = (3x – 7)/8
By giving different values to x, we obtain the corresponding values of y

x	1	5	-3
y	- ½	1	-2

Now plot the points (1, -1/2), (5, 1) and (-3, -2) on the graph and join them to obtain a line.

2x – y + 4 = 0
It can be written as
2x = y – 4
x = (y – 4)/ 2
By giving different values to y, we obtain the corresponding values of x

x	-2	-1	0
y	0	2	4

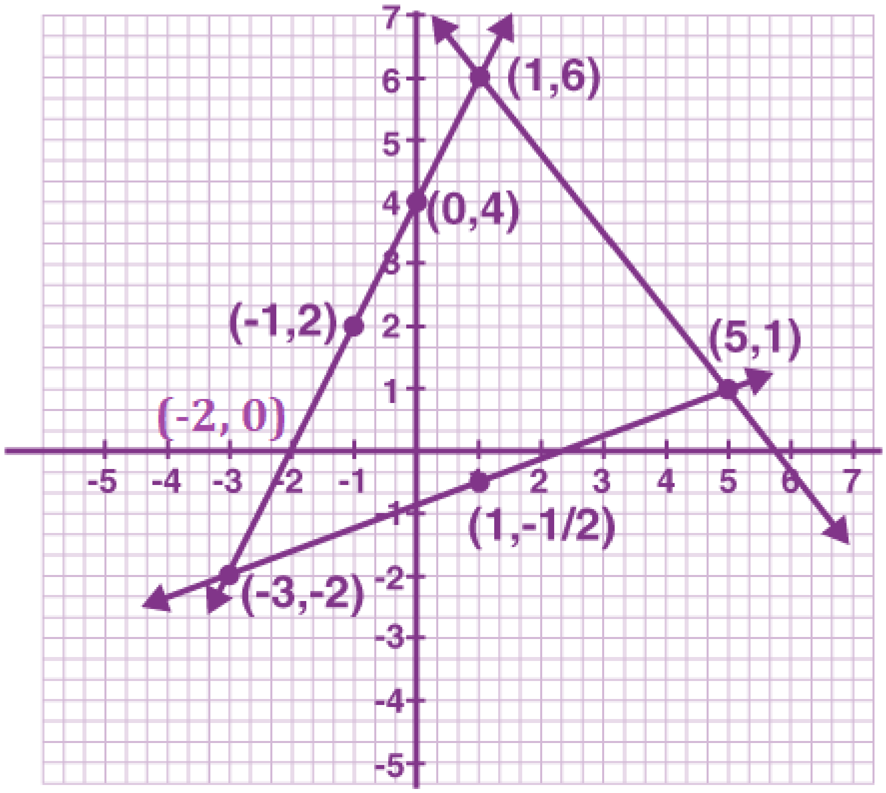
Now plot the points (-2, 0), (-1, 2) and (0, 4) on the graph and join them to obtain another line.

5x + 4y = 29
It can be written as
5x = 29 – 4y
x = (29 – 4y)/ 5

x	5	1	-4
y	1	6	9

Now plot the points (5, 1), (1, 6) and (-4, 9) on the graph and join them to obtain another line.
These three lines intersect each other at (-3, -2), (1, 5) and (1, 6)

Hence, the coordinates of the vertices of the triangle formed by these lines are (-3, -2), (1, 5) and (1, 6).



9. Find graphically the coordinates of the vertices of the triangle formed by the lines y – 2 = 0, 2y + x = 0 and y + 1 = 3 (x – 2). Hence, find the area of the triangle formed by these lines.
Solution:

y – 2 = 0
It can be written as
y = 2 which is parallel to x-axis

x	0	1	3
y	2	2	2

2y + x = 0
It can be written as
x = - 2y

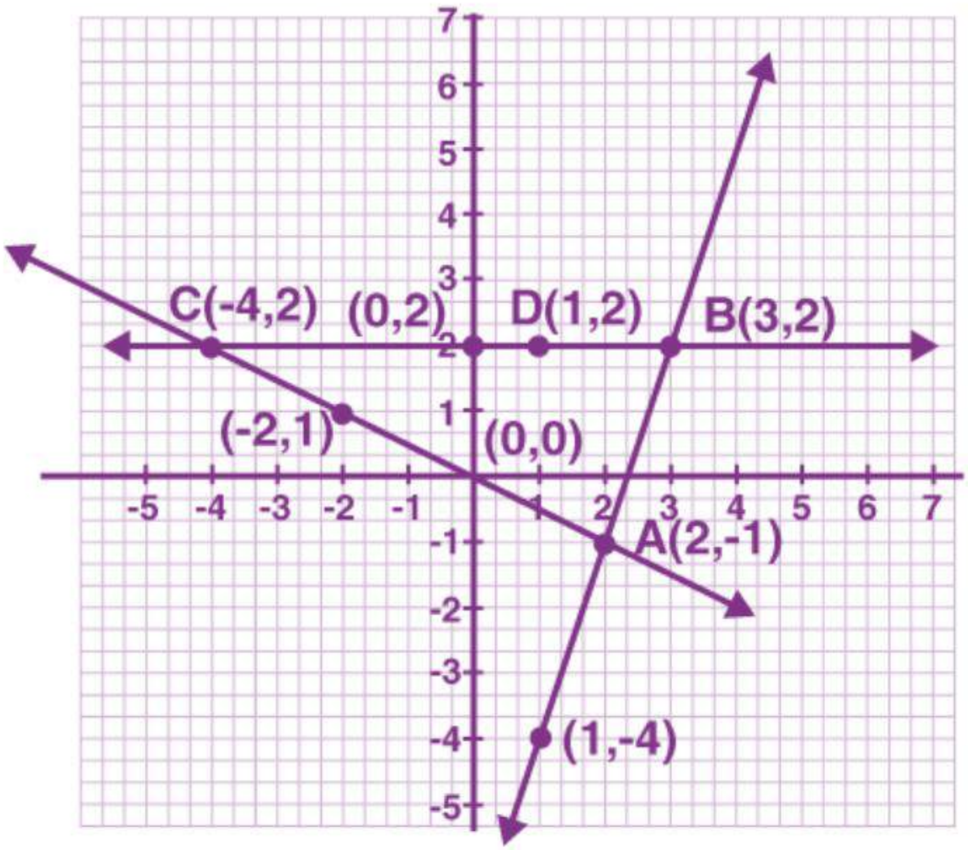
x	0	-2	-4
y	0	1	2

Now plot the points (0, 0), (-2, 1) and (-4, 2) on the graph and join them to obtain a line.

y + 1 = 3 (x – 2)
It can be written as
y + 1 = 3x - 6
By giving different values to x, we obtain the corresponding values of y

x	1	2	3

y	-4	-1	2



Now plot the points (-2, 0), (-1, 2) and (0, 4) on the graph and join them to obtain another line.
We can see that the three lines intersect each other
Now the coordinates of the vertices of the triangle are (2, 1), (3, 2), (4, -2) and
Area of triangle = (BC × AB)/2
Substituting the values
= (7 × 3)/ 2
= 21/2
= 10.5 cm^2

10. A line segment is of length 10 units and one of its end is (-2, 3). If the ordinate of the other end is 9, find the abscissa of the other end.
Solution:

Ordinates of the point on the other end (y) = 9
Consider abscissa = x
Distance between the two ends (-2, 3) and (x, 9) = $\sqrt{(x + 2)^2 + (9 - 3)^2}$
$\sqrt{(x + 2)^2 + 6^2} = 10$

Squaring on both sides
$x^2 + 4x + 4 + 36 = 100$
By further calculation
$x^2 + 4x = 100 – 36 – 4$
$x^2 + 4x – 60 = 0$
It can be written as
$x^2 + 10x – 6x – 60 = 0$
$x (x + 10) – 6 (x + 10) = 0$
$(x + 10) (x – 6) = 0$
Here
$x + 10 = 0$
So we get
$x = - 10$
Similarly
$x – 6 = 0$
$x = 6$

Therefore, abscissa of the other end is – 10 or 6.

11. A (-4, -1), B (-1, 2) and C (α, 5) are the vertices of an isosceles triangle. Find the value of α given that AB is the unequal side.
Solution:

It is given that
A (-4, -1), B (-1, 2) and C (α, 5) are the vertices of an isosceles triangle
AB is the unequal side
AC = BC

We know that
$AC = \sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$
Substituting the values
$AC = \sqrt{(\alpha + 4)^2 + (5 + 1)^2}$
$AC = \sqrt{(\alpha + 4)^2 + 6^2}$

$BC = \sqrt{(\alpha + 1)^2 + (5 - 2)^2}$
By further calculation
$BC = \sqrt{(\alpha + 1)^2 + 3^2}$

So we get
$\sqrt{(\alpha + 4)^2 + 6^2} = \sqrt{(\alpha + 1)^2 + 3^2}$
By squaring on both sides
$(\alpha + 4)^2 + 6^2 = (\alpha + 1)^2 + 3^2$
Now expanding using formula

$\alpha^2 + 8\alpha + 16 + 36 = \alpha^2 + 2\alpha + 1 + 9$
By further calculation
$8\alpha - 2\alpha = 1 + 9 - 16 - 36$
So we get
$6\alpha = -42$
$\alpha = -42/6 = -7$

Therefore, the value of α is – 7.

12. If A (-3, 2), B (α, β) and C (-1, 4) are the vertices of an isosceles triangle, prove that $\alpha + \beta = 1$, given AB = BC.
Solution:

It is given that
A (-3, 2), B (α, β) and C (-1, 4) are the vertices of an isosceles triangle
AB = BC

Here
$AB = \sqrt{(\alpha + 3)^2 + (\beta - 2)^2}$
$BC = \sqrt{(\alpha + 1)^2 + (\beta - 4)^2}$

Now
AB = BC
$\sqrt{(\alpha + 3)^2 + (\beta - 2)^2} = \sqrt{(\alpha + 1)^2 + (\beta - 4)^2}$
By squaring on both sides
$(\alpha + 3)^2 + (\beta - 2)^2 = (\alpha + 1)^2 + (\beta - 4)^2$
Expanding using the formula
$\alpha^2 + 6\alpha + 9 + \beta^2 - 4\beta + 4 = \alpha^2 + 2\alpha + 1 + \beta^2 - 8\beta + 16$
By further calculation
$6\alpha - 2\alpha - 4\beta + 8\beta = 16 - 9 - 4 + 1$
$4\alpha + 4\beta = 4$
Dividing by 4
$\alpha + \beta = 1$

Therefore, it is proved.

13. Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.
Solution:

Consider points A (3, 0), B (6, 4) and C (-1, 3) are the vertices of a right angled isosceles triangle.
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
$= \sqrt{(6 - 3)^2 + (4 - 0)^2}$
By further calculation
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$

BC = $\sqrt{(-1 - 6)^2 + (3 - 4)^2}$
By further calculation
= $\sqrt{(-7)^2 + (-1)^2}$
So we get
= $\sqrt{49 + 1}$
= $\sqrt{50}$
= $\sqrt{(25 \times 2)}$
= $5\sqrt{2}$

AC = $\sqrt{(-1 - 3)^2 + (3 - 0)^2}$
By further calculation
= $\sqrt{(-4)^2 + 3^2}$
So we get
= $\sqrt{16 + 9}$
= $\sqrt{25}$
= 5

Here
$AB^2 + AC^2 = 5^2 + 5^2$
So we get
= 25 + 25
= 50
= BC^2

Therefore, it is proved.

14. (i) Show that the points (2, 1), (0, 3), (-2, 1) and (0, -1), taken in order, are the vertices of a square. Also find the area of the square.
(ii) Show that the points (-3, 2), (-5, -5), (2, -3) and (4, 4), taken in order, are the vertices of rhombus. Also find its area. Do the given points from a square?
Solution:
(i) Consider A (2, 1), B (0, 3), C (2, -1) and D (0, -1) taken in order are the vertices of the square
AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
= $\sqrt{(0 - 2)^2 + (3 - 1)^2}$
= $\sqrt{2^2 + 2^2}$
So we get
= $\sqrt{4 + 4}$
= $\sqrt{8}$

BC = $\sqrt{(-2 - 0)^2 + (1 - 3)^2}$
By further calculation
= $\sqrt{(-2)^2 + (-2)^2}$
So we get
= $\sqrt{4 + 4}$
= $\sqrt{8}$

CD = $\sqrt{(0 - 2)^2 + (-1 - 1)^2}$
By further calculation
= $\sqrt{2^2 + 2^2}$
So we get
= $\sqrt{4 + 4}$
= $\sqrt{8}$

CA = $\sqrt{(2 - 0)^2 + (1 + 1)^2}$
By further calculation
= $\sqrt{2^2 + 2^2}$
So we get
= $\sqrt{4 + 4}$
= $\sqrt{8}$

DA = $\sqrt{(2 - 0)^2 + (1 + 1)^2}$
By further calculation
= $\sqrt{2^2 + 2^2}$
So we get
= $\sqrt{4 + 4}$
= $\sqrt{8}$
AB = BC = CD = DA

Here ABCD is a square with side $\sqrt{8}$
Area = side^2 = $(\sqrt{8})^2$ = 8 sq. units

(ii) Consider A (-3, 2), B (-5, -5), C (2, -3) and D (4, 4) taken in order are the vertices of a rhombus
AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
= $\sqrt{(-5 + 3)^2 + (-5 - 2)^2}$
= $\sqrt{(-2)^2 + (-7)^2}$
So we get
= $\sqrt{4 + 49}$
= $\sqrt{53}$
BC = $\sqrt{(2 + 5)^2 + (-3 + 5)^2}$
By further calculation
= $\sqrt{7^2 + 2^2}$
So we get
= $\sqrt{49 + 4}$
= $\sqrt{53}$

CD = $\sqrt{(4 - 2)^2 + (4 + 3)^2}$
By further calculation
= $\sqrt{2^2 + 7^2}$
So we get
= $\sqrt{4 + 49}$
= $\sqrt{53}$

DA = $\sqrt{(-3-4)^2 + (2-4)^2}$
By further calculation
= $\sqrt{(-7)^2 + (-2)^2}$
So we get
= $\sqrt{4 + 49}$
= $\sqrt{53}$
AB = BC = CD = DA
Here ABCD is a square or rhombus

Diagonal AC = $\sqrt{(2+3)^2 + (-3-2)^2}$
By further calculation
= $\sqrt{5^2 + 5^2}$
So we get
= $\sqrt{25 + 25}$
= $\sqrt{50}$

BD = $\sqrt{(4+5)^2 + (4+5)^2}$
By further calculation
= $\sqrt{9^2 + 9^2}$
So we get
= $\sqrt{81 + 81}$
= $\sqrt{162}$
AC $\neq$ BD
So ABCD is a rhombus not a square

We get
Area = Product of diagonal/2
Substituting the values
= $\sqrt{50} \times \sqrt{162}/ 2$
= $\sqrt{(8100/2)}$
So we get
= 90/2
= 45 sq. units

15. The ends of a diagonal of a square have co-ordinates (-2, p) and (p, 2). Find p if the area of the square is 40 sq. units.
Solution:

It is given that
Ends of a diagonal of a square (-2, p) and (p, 2)
Area of square = 40 sq. units
Side = $\sqrt{40}$ units = 2 $\sqrt{10}$ units
Diagonal = $\sqrt{2} \times$ side
Substituting the values
= $\sqrt{2} \times \sqrt{40}$
So we get
= $\sqrt{80}$

$= 4\sqrt{5}$ unit

Diagonal AC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
$\sqrt{(p + 2)^2 + (2 - p)^2} = 4\sqrt{5}$
By squaring on both sides
$(p + 2)^2 + (2 - p)^2 = 16 \times 5 = 80$
Expanding using formula
$p^2 + 4p + 4 + 4 - 4p + p^2 = 80$
By further calculation
$2p^2 + 8 = 80$
$2p^2 = 80 - 8 = 72$
So we get
$p^2 = 72/2 = 36 = (\pm 6)^2$
$p = \pm 6$

Therefore, the value of p is (6, -6).

16. What type of quadrilateral do the points A (2, -2), B (7, 3), C (11, -1) and D (6,-6), taken in that order, form?
Solution:

We know that
A (2, -2), B (7, 3), C (11, -1) and D (6,-6), taken in that order are the vertices of a quadrilateral ABCD

AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
$= \sqrt{(7 - 2)^2 + (3 + 2)^2}$
By further calculation
$= \sqrt{5^2 + 5^2}$
$= \sqrt{25 + 25}$
$= \sqrt{50}$
BC $= \sqrt{(11 - 7)^2 + (-1 - 3)^2}$
By further calculation
$= \sqrt{4^2 + (-4)^2}$
So we get
$= \sqrt{16 + 16}$
$= \sqrt{32}$

CD $= \sqrt{(6 - 11)^2 + (-6 + 1)^2}$
By further calculation
$= \sqrt{(-5)^2 + (-5)^2}$
So we get
$= \sqrt{25 + 25}$
$= \sqrt{50}$

DA $= \sqrt{(6 - 2)^2 + (-6 + 2)^2}$
By further calculation

$= \sqrt{4^2 + (-4)^2}$
So we get
$= \sqrt{16 + 16}$
$= \sqrt{32}$

Here AB = CD and BC = DA
Hence, ABCD is a rectangle as the opposite sides are equal.

17. Find the coordinates of the centre of the circle passing through the three given points A (5, 1), B (-3, -7) and C (7, -1).
Solution:

Consider (x, y) as the coordinates of the centre of the circle
Points A (5, 1), B (- 3, -7) and C (7, -1) are on the circle
OA = OB = OC

$OA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting the values
$= \sqrt{(x - 5)^2 + (y - 1)^2}$

$OB = \sqrt{(x + 3)^2 + (y + 7)^2}$

$OC = \sqrt{(x - 7)^2 + (y + 1)^2}$

Here $OA^2 = OB^2$ and $OA^2 = OC^2$
Now by equating both
$(x - 5)^2 + (y - 1)^2 = (x + 3)^2 + (y + 7)^2$
Expanding using the formulas
$x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 6x + 9 + y^2 + 14y + 49$
By further calculation
$6x + 14y + 10x + 2y = - 9 - 49 + 25 + 1$
So we get
$16x + 16y = - 32$
Divide by 16
$x + y = - 2$
$x = - 2 - y$ …… (1)

$OA^2 = OC^2$
Substituting the values
$(x - 5)^2 + (y - 1)^2 = (x - 7)^2 + (y + 1)^2$
Expanding using the formulas
$x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 - 14x + 49 + y^2 + 1 + 2y$
By further calculation
$- 10x + 14x - 2y - 2y = 49 + 1 - 25 - 1$
So we get
$4x - 4y = 24$
Dividing by 4

$x - y = 6$ …… (2)

Substituting the value of (1) in (2)
$(-2 - y) - y = 6$
$-2 - y - y = 6$
By further calculation
$- 2y = 6 + 2$
$y = -8/2 = -4$

Substituting the value of y in equation (1)
$x = - 2 - y$
$x = - 2 - (-4)$
So we get
$x = - 2 + 4 = 2$

Therefore, the coordinates of the centre of the circle are (2, -4).