Maths chapter-10(triangles)

Note- See diagrams in your book

Exercise 10.1

1.It is given that ΔABC ≅ ΔRPQ. Is it true to say that BC = QR ?Why?

Solution:

Given ΔABC ≅ ΔRPQ

Therefore their corresponding sides and angles are equal.

Therefore BC = PQ

Hence it is not true to say that BC = QR

2. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statementtrue? Why?

Solution:

No, it is not true statement as the angles should be included angle of there two given sides.

3. In the given figure, AB=AC and AP=AQ. Provethat

ΔAPC ≅ΔAQB

CP =BQ

∠APC =∠AQB.

Solution:

In △ APC and △AQB AB=AC and AP=AQ[given]

From the given figure, ∠A = ∠A [common in both the triangles]

Therefore, using SAS axiom we have ΔAPC ≅ ΔAQB

In △ APC and △AQB AB=AC and AP=AQ[given]

From the given figure, ∠A = ∠A [common in both the triangles]

By using corresponding parts of congruent triangles concept we have BQ = CP

In △ APC and △AQB AB=AC and AP=AQ[given]

From the given figure, ∠A = ∠A [common in both the triangles]

By using corresponding parts of congruent triangles concept we have

∠APC = ∠AQB.

4. In the given figure, AB = AC, P and Q are points on BA and CA respectively such that AP = AQ. Provethat

ΔAPC ≅ΔAQB

CP =BQ

∠ACP =∠ABQ.

Solution:

In the given figure AB =AC

P and Q are point on BA and CA produced respectively such that AP = AQ

Now we have to prove ΔAPC ≅ ΔAQB

By using corresponding parts of congruent triangles concept we have CP = BQ

∠ACP = ∠ABQ

CP =BQ

∠ACP = ∠ABQ In Δ APC and ΔAQB AC = AB(Given)

AP = AQ(Given)

∠PAC =∠QAB (Vertically opposite angle)

.

5. In the given figure, AD = BC and BD = AC. Prove that:

<ADB = ∠BCA and ∠DAB = ∠CBA

Solution:

Given: in the figure, AD = BC, BD = AC

To prove :

∠ADB =∠BCA

∠DAB =∠CBA

Proof : in ΔADB and ΔACB

AB = AB (Common) AD = BC(given)

DB = AC(Given)

ΔADB = ΔACD (SSS axiom)

∠ADB =∠BCA

∠DAB =∠CBA

6.In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB =∠CBA.

Prove that

ΔABD ≅ ΔBAC

BD =AC

∠ABD =∠BAC.

Solution:

Given : in the figure ABCD is a quadrilateral In which AD = BC

∠DAB = ∠CBA

To prove :

ΔABD =ΔBAC

∠ABD =∠BAC

Proof : in ΔABD and ΔABC

AB = AB (common)

∠DAB = ∠CBA (Given)

AD = BC

ΔABD ≅ ΔABC (SASaxiom)

BD =AC

(ii) ∠ABD =∠BAC

7. In the given figure, AB = DC and AB || DC. Prove that AD =BC.

Solution :

Given: in the given figure.

AB = DC, AB ∥ DC

To prove : AD = BC Proof : AB ∥ DC

∠ABD = ∠CDB (Alternate angles)

In ΔABD and ΔCDB

AB = DC

∠ABD = ∠CDB (Alternate angles) BD = BD (common)

ΔABD ≅ CDB (SAS axiom) AD = BC

8. In the given figure. AC = AE, AB = AD and ∠BAD = ∠CAE. Show that BC =DE.

Solution:

Given: in the figure, AC = AE, AB = AD

∠BAD = ∠CAE

To prove : BC = DE

Proof : in ΔABC and ΔADE

AB = AD (given) AC = AE (given)

∠BAD + ∠DAC + ∠CAE

∠BAC = ∠DAE

ΔABC = Δ ADE (SAS axiom)

BC = DE

9. In the adjoining figure, AB = CD, CE = BF and ∠ACE = ∠DBF. Provethat

i).ΔACE ≅ΔDBF

ii).AE =DF.

Solution:

Given : in the given figure AB = CD

CE = BF

∠ACE = ∠DBF

To prove : (i) ΔACE ≅ ΔDBF

ΔACE ≅ ΔDBF (SASaxiom)

AE = DF

AE = DF Proof : AB =CD

Adding BC to both sides AB + BC = BC + CD

AC = BD

Now in ΔACE and ΔDBF

AC = BD (Proved) CE = BF (Given)

∠ACE = ∠DBF (SAS axiom)

Exercise 10.2

1.In triangles ABC and PQR, ∠A= ∠Q and ∠B = ∠R. Which side of APQR should be equal to side AB of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In triangle ABC and triangle PQR

∠A = ∠Q

∠B =∠R

AB = QP

Because triangles are congruent of their corresponding two angles and included sides are equal

2.In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of APQR should be equal to side BC of AABC so that the two triangles are congruent? Give reason foryour answer.

Solution:

In ΔABC and ΔPQR

∠A = ∠Q

∠B = ∠R

Their included sides AB and QR will be equal for their congruency. Therefore, BC = PR by corresponding parts of congruent triangles.

3.“If two angles and a side of one triangle are equal to two angles and a sideof another triangle, then the two triangles must be congruent”. Is the statement true? Why?

Solution:

The given statement can be true only if the corresponding (included) sides are equal otherwise not.

4.In the given figure, AD is median of ΔABC, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN. Solution:

Given in ΔABC, AD is median BM and CN are perpendicular to AD form B and C respectively.

To prove:

BM = CN

Proof:

In ΔBMD and ΔCND

BD = CD (because AD is median)

∠M = ∠N

∠BDM = ∠CDN (vertically opposite angles)

ΔBMD ≅ ΔCND (AAS axiom) Therefore, BM = CN.

5. In the given figure, BM and DN are perpendiculars to the line segment AC. If BM = DN, prove that AC bisectsBD.

Solution:

Given in figure BM and DN are perpendicular to AC BM =DN

To prove:

AC bisects BD that is BE = ED Construction:

Join BD which intersects Ac at E Proof:

In ΔBEM and ΔDEN BM = DN

∠M = ∠N (given)

∠DEN = ∠BEM (vertically opposite angles)

ΔBEM ≅ ΔDEN BE = ED

Which implies AC bisects BD

6.In the given figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC ≅ ΔCDA.

Solution:

In the given figure, two lines l and m are parallel to each other and lines p and q are also a pair of parallel lines intersecting each other at A, B, C and D. AC is joined.

To prove:

ΔABC ≅ ΔCDA Proof:

In ΔABC and ΔCDA AC = AC (common)

∠ACB = ∠CAD (alternateangles)

∠BAC = ∠ACD (alternateangles)

ΔABC ≅ Δ DCA (ASA axiom)

7.In the given figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line segments AB and CD.

Solution:

In the given figure, lines AB and CD intersect each other at O such that BC ∥ AD and BC = DA

To prove:

O is the midpoint of Ab and Cd Proof:

Consider ΔAOD and ΔBOC AD = BC (given)

∠OAD = ∠OBC (alternateangles)

∠ODA = ∠OCB (alternateangles)

ΔAOD ≅ ΔBOC (SAS axiom) Therefore, OA = OB and OD =OC

Therefore O is the midpoint of AB and CD.

Exercise 10.3

1.ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C. Solution:

In right angled triangle ABC, ∠A = 90o

∠B + ∠C = 180o- ∠A

= 180o– 90o= 90o

Because AB = AC

∠C = ∠B (Angles opposite to equal sides)

∠B + ∠B = 90o(2∠B = 90o

∠B = 90/2o= 45o

∠B = ∠C =45o

∠B = ∠C =45o

2.Show that the angles of an equilateral triangle are 60°each.

Solution:

ΔABC is an equilateral triangle

AB = BC =CA

∠A = ∠B = ∠C (opposite to equal sides)

But ∠A + ∠B + ∠C = 180o(sum of angles of a triangle) 3∠A = 180o(∠A = 180o/3 = 60o)

∠A = ∠B = ∠C = 60o

3.Show that every equiangular triangle isequilateral.

Solution:

ΔABC is an equiangular

∠A = ∠B = ∠C

In ΔABC

∠B =∠C

AC = AB (sides opposite to equal angles) Similarly, ∠C = ∠A

BC = AB

From (i) and (ii) AB = BC = AC

ΔABC is an equilateral triangle

4.In the following diagrams, find the value ofx:

Solution:

in following diagram given that AB=AC

That is ∠B = ∠ ACB (angles opposite to equal sides in a triangle are equal) In a triangle are equal)

Now, ∠A + ∠B + ∠ACB = 180o

(sum of all angles in a triangle is 180o)

50 + ∠B + ∠B = 180o

(∠A = 50o(given) ∠B = ∠ACB)

50o+ 2 ∠B = 180o(2 ∠B = 180o– 50o

2 ∠B = 130o(∠B = 130/2 = 65o

∠ACB = 65o

Also ∠ACB + xo= 180o(Linear pair) 65o+ xo= 180o(xo= 180o– 65o

xo= 115o

Hence, Value of x = 115

inΔPRS,

Given that PR = RS

∠PSR = ∠RPS

(Angles opposite in a triangle, equal sides are equal)

30o= ∠RPS (∠PPS = 30o……(1)

∠QPS = ∠QPR + ∠RPS

∠QPS = 52o+ 30o

(Given, ∠QPR = 52oand from (i), ∠RPS = 30o

∠QPS = 82o

Now, In ΔPQS

∠QPS + ∠QSP + PQS = 180o

(sum of all angles in a triangles is 180o)

= 82o+ 30o+ xo= 180o

(from (2) ∠QPS = 82oand ∠QSP = 30o(given)

112o+ xo= 180o(xo= 180o–1120

Hence, Value of x = 68

In the following figure, Given That, BD = CD = AC and ∠DBC = 27oNow in ΔBCD

BD = CD (Given)

∠DBC = ∠BCD …….(1)

(in a triangle sides opposite equal angles are equal) Also,, ∠DBC=27o (given) (2)

From (1) and (2) we get

∠BCD = 27o

Now, ext ∠CDA = ∠DBC + ∠BCD

(exterior angles is equal to sum of two interior opposite angles) Ext ∠CDA = 27o+ 270(from (2) and (3)

∠CDA =54o (from(4)) (5)

Also, in ΔACD

∠CAD + ∠CDA + ∠ACD = 1800

(sum of all angles in a triangle is 180o) 54O+ 54O+ Y = 180O

108O+ Y = 180O(Y = 180O– 108O

y = 72o

8.(a) In the figure (1) given below, ABC is an equilateral triangle. Base BC isproduced

to E, such that BC'= CE. Calculate ∠ACE and ∠AEC.

8(ii).In the figure (2) given below, prove that ∠ BAD : ∠ ADB = 3 :1.

8.(iii).In the figure (3) given below, AB || CD. Find the values of x, y and z.

Solution:

in followingfigure

Given. ABC is an equilateral triangle BC = CE To find. ∠ACE and ∠AEC

As given that ABC is an equilateral triangle, That is ∠BAC = ∠B = ∠ACB=600 (1)

(each angle of an equilateral triangle is 60o)

Now, ∠ACE = ∠BAC + CB

(Exterior angle is equal to sum of two interior opposite angles) (∠ACE = 60o+ 60o)

10. In the given figure, AD, BE and CF arc altitudes of ΔABC. If AD = BE = CF, prove that ABC is an equilateraltriangle.

Given : in the figure given,

AD, BE and CF are altitudes of ΔABC and

AD = BE = CF

To prove : ΔABC is an equilateral triangle Proof: in the right ΔBEC and ΔBFC Hypotenuse BC = BC (Common)

Side BE = CF (Given)

ΔBEC ≅ ΔBFC (RHS axiom)

∠C = ∠B

AB = AC (sides opposite to equal angles)

Similarly we can prove that ΔCFA ≅ ΔADC

∠A = ∠C AB = BC

From (i) and (ii) AB = BC = AC

ΔABC is an equilateral triangle

9.In the given figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that DE = DF. Prove that ABC is an isoscelestriangle.

Solution:

In triangle ABC

D is the midpoint of BC DE perpendicular to AB

And DF perpendicular to AC DE = DE

To prove:

Triangle ABC is an isosceles triangle Proof:

In the right angled triangle BED and CDF Hypotenuse BD = DC (because D is a midpoint)

Side DF = DE (given)

$\Delta BED \cong \Delta CDF$ (RHS axiom)

$\angle C = \angle B$

AB = AC (sides opposite to equal angles)

ΔABC is an isosceles triangle

Exercise 10.4

1.In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is longest? Give reason for your answer.

Solution:

In ΔPQR, ∠P = 70o, ∠R = 30oBut ∠P + ∠Q + ∠R = 180o100o+ ∠Q = 180o

∠Q = 180o– 100o= 180o

∠Q = 80othe greatest angle

Its opposite side PR is the longest side (side opposite to greatest angle is longest)

2.Show that in a right angled triangle, the hypotenuse is the longestside.

Solution:

Given: in right angled ΔABC, ∠B = 90o

To prove: AC is the longest side Proof : in ΔABC,

∠B = 90o

∠A and ∠C are acute angles That is less then 90o

∠B is the greatest angle Or ∠B> ∠C and ∠B> ∠A AC > AB and AC > BC

Hence AC is the longest side

3.PQR is a right angle triangle at Q and PQ : QR = 3:2. Which is the least angle.

Solution:

Here, PQR is a right angle triangle at Q. Also given that PQ : QR = 3:2

Let PQ = 3x, then, QR = 2x

It is clear that QR is the least side,

Then, we know that the least angle has least side Opposite to it.

Hence ∠P is the least angle

4.In Δ ABC, AB = 8 cm, BC = 5.6 cm and CA = 6.5 cm. Which is (i) the greatest angle? (ii) the smallest angle ?

Solution:

Given that AB = 8 cm, BC = 5.6 cm, CA = 6.5 cm. Here AB is the greatest side

Then ∠C is the least angle

The greatest side has greatest angle opposite to it) Also, BC Is the least side

Then ∠A is the least angle

(the least side has least angle opposite to it)